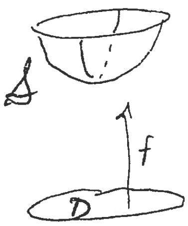
Common Parameterizations

Graphs

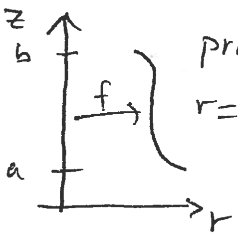


$$\vec{r}(x,y) = (x, y, f(x,y)) \qquad d\vec{S} = \vec{n}\,dS = (-f_x, -f_y, 1)\,dx\,dy$$

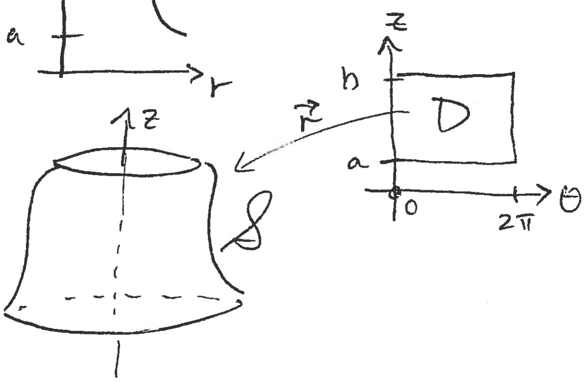
$$\iint_{\mathcal{S}} (P,Q,R)\cdot\vec{n}\,dS = \iint_D -Pf_x - Qf_y + R\,dx\,dy$$

$$\iint_{\mathcal{S}} g\,dS = \iint_D g\sqrt{f_x^2+f_y^2+1}\;dx\,dy$$

Surfaces of Revolution



profile function
$r = f(z) \geq 0$



$$\vec{r}(z,\theta) = (f(z)\cos\theta,\ f(z)\sin\theta,\ z)$$

$$d\vec{S} = \vec{n}\,dS = (f(z)\cos\theta,\ f(z)\sin\theta,\ -f(z)f'(z))\,dz\,d\theta$$

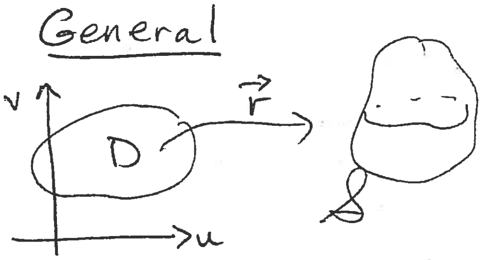
$$= (x, y, -ff')\,dz\,d\theta$$

$$dS = f(z)\sqrt{1+f'(z)^2}\;dz\,d\theta$$

$$\iint_{\mathcal{S}} (P,Q,R)\cdot\vec{n}\,dS = \iint_D f(z)\left(P\cos\theta + Q\sin\theta - Rf'(z)\right)dz\,d\theta$$

$$\iint_{\mathcal{S}} g\,dS = \iint_D g f\sqrt{1+(f')^2}\;dz\,d\theta$$

General



$$\vec{r} : D \to \mathbb{R}^3$$

$$d\vec{S} = \vec{n}\,dS = \vec{r}_u \times \vec{r}_v\,du\,dv = \frac{\partial\vec{r}}{\partial u}\times\frac{\partial\vec{r}}{\partial v}\,du\,dv$$

$$dS = |\vec{r}_u \times \vec{r}_v|\,du\,dv$$

$$\iint_{\mathcal{S}} \vec{F}\cdot\vec{n}\,dS = \iint_D \vec{F}\cdot(\vec{r}_u\times\vec{r}_v)\,du\,dv$$

$$\iint_{\mathcal{S}} g\,dS = \iint_D g\,|\vec{r}_u\times\vec{r}_v|\,du\,dv$$

Surfaces

Parameterize

$$\vec{r}(u,v) = (x(u,v),\ y(u,v),\ z(u,v)) \in \mathbb{R}^3$$

$$d\vec{S} = \vec{n}\,dS = \vec{r}_u \times \vec{r}_v \ du\,dv$$

$$\vec{n} = \vec{r}_u \times \vec{r}_v \Big/ |\vec{r}_u \times \vec{r}_v|$$

$$dS = |\vec{r}_u \times \vec{r}_v|\,du\,dv$$

1. Graph of $z = f(x,y)$

$$\vec{r}(x,y) = (x,\ y,\ f(x,y))$$

$$d\vec{S} = (-f_x,\ -f_y,\ 1)\ dx\,dy$$

2. Surface of revolution $\quad r = f(z)$

$$\vec{r}(\theta,z) = (f(z)\cos\theta,\ f(z)\sin\theta,\ z)$$

$$d\vec{S} = f(z)\,(\cos\theta,\ \sin\theta,\ -f'(z))\ d\theta\,dz$$

3. Part of sphere $\rho = R_1$, constant

$$\vec{r}(\theta,\phi) = R_1(\sin\phi\cos\theta,\ \sin\phi\sin\theta,\ \cos\phi)$$

$$d\vec{S} = R_1^2 \sin\phi\,(\sin\phi\cos\theta,\ \sin\phi\sin\theta,\ \cos\phi)\ d\theta\,d\phi$$

$$= R_1 \sin\phi\ (x,y,z)\,d\theta\,d\phi$$

Parameterization of S	$n\,dS$	$\int_S F\cdot n\,dS = \int_S (P,Q,R)\cdot n\,dS$	$\int_S f(p)\,dS$
$p(x,y) = (x,y,f(x,y))$	$(-f_x, -f_y, 1)\,dx\,dy$	$\int_R -Pf_x - Qf_y + R \; dx\,dy$	$\int_R f(p)\sqrt{f_x^2+f_y^2+1}\; dx\,dy$
project $g(x,y,z)=0$ onto (x,y)	$\frac{\nabla g}{\lvert g_z\rvert}\,dx\,dy = \frac{(g_x,g_y,g_z)}{\lvert g_z\rvert}\,dx\,dy$	$\int_R \frac{Pg_x+Qg_y+Rg_z}{\lvert g_z\rvert}\,dx\,dy$	$\int_R f(p)\frac{\sqrt{g_x^2+g_y^2+g_z^2}}{\lvert g_z\rvert}\,dx\,dy$
to project onto (x,z)	change $\lvert g_z\rvert$ to $\lvert g_y\rvert$ (the missing variable in (x,z)), and $dx\,dy$ to $dx\,dz$		
similarly for other	coordinate pairs.		
surface of revolution $r=f(z)$; $p(z,\theta)=(f(z)\cos\theta, f(z)\sin\theta, z)$	$(f(z)\cos\theta, f(z)\sin\theta, -f(z)f'(z))\,dz\,d\theta$ $= (x,y,-ff')\,dz\,d\theta$	$\int_R Px+Qy - Rff' \; dz\,d\theta$ $= \int_R r(z)(P\cos\theta + Q\sin\theta - Rr')\,dz\,d\theta$	$\int_R f(p)\,\lvert r(z)\rvert\sqrt{1+r'(z)^2}\; dz\,d\theta$
S in a sphere $\rho=R_1$; $p(\phi,\theta)=(R_1\sin\phi\cos\theta, R_1\sin\phi\sin\theta, R_1\cos\phi)$	$R_1^2\sin\phi(\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$	$\int_R (Px+Qy+Rz)R_1\sin\phi\; d\phi\,d\theta$	$\int_R f(p)R_1^2\sin\phi\; d\phi\,d\theta$
General $p(u,v)=(x,y,z)$	$\left(\frac{\partial p}{\partial u}\times\frac{\partial p}{\partial v}\right)du\,dv$	$\int_R (P,Q,R)\cdot\left(\frac{\partial p}{\partial u}\times\frac{\partial p}{\partial v}\right)du\,dv$	$\int_R f(p(u,v))\left\lvert\frac{\partial p}{\partial u}\times\frac{\partial p}{\partial v}\right\rvert du\,dv$