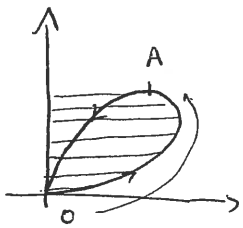
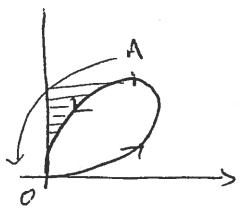
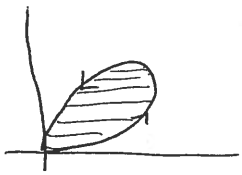
$\int_C x\,dy$ says to integrate distance x from the y-axis to the curve in the y-direction. Break the process into two steps: where dy is positive & where it is negative.



From O to A
$dy > 0$
so $\int x\,dy$ is the shaded area.

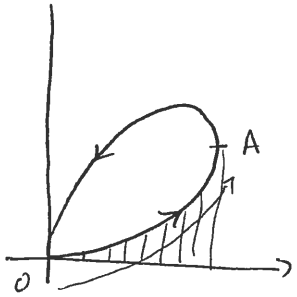


From A back to O
$dy < 0$
so $\int x\,dy$ is the negative of the shaded area.

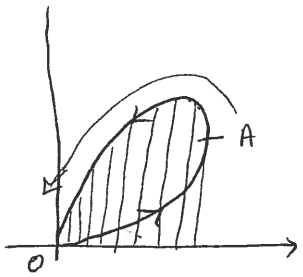


Adding these, we get exactly the shaded area:

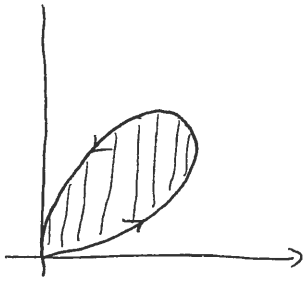
$\int_C y\,dx$ similarly



From O to A
$dx > 0$ so
$\int y\,dx$ is the shaded area.



From A back to O
$dx < 0$
so $\int y\,dx$ is the negative of the shaded area.



Adding these, we get exactly the negative of the shaded area.