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Math 2030, Winter 2005, Test 3
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1. Draw the diagram which shows the relation between (x, y) and (r, θ). Write the formulas for x and y in terms of r and θ, for $dA = dx\,dy$ in terms of r and θ, and for $dV = dx\,dy\,dz$ in terms of r, θ and z.

2. Draw the diagram which shows the relation between (r, z) and (ρ, ϕ). Write the formulas for r and z in terms of ρ and ϕ, for $dA = dr\,dz$ in terms of ρ and ϕ, and for $dV = dx\,dy\,dz$ in terms of ρ, ϕ, and θ.

3. Suppose that $1 \leq f(x, y) \leq 1 + y^2$ for all (x, y). What can you say about $\iint_R f\,dA$ if R is the rectangle $[0, 2] \times [0, 3]$?

4. Find the area and centroid of the region below $y = 1 - x^2$ in the first quadrant.

5. Let R be the part of the disk $x^2 + y^2 \leq 1$ which lies between $\theta = \pi/6$ and $\theta = \pi/4$. Find $\bar{x}$, the x-coordinate of the centroid of R.

6. Let $(x, y) = F(u, v) = (u^3 v, v^3 - u)$.

   (a) Find the Jacobian of F,
   $$\frac{\partial(x, y)}{\partial(u, v)}$$

   (b) If R is the rectangle $[0, 1] \times [0, 1]$ in the uv-plane, find the area of its image, $F(R)$.

7. Let R be the part of the cylinder $0 \leq r \leq 1$ which lies between the xy-plane and the paraboloid $z = 1 + r^2$. Find the volume of R.

8. Let R be the same region as in the preceding problem. Find the moment of inertia of R about the z-axis.

9. Let B be the part of the ball $0 \leq \rho \leq 1$ which lies inside the cone $0 \leq \phi \leq \pi/4$. Find the volume of B.

10. Let B be the same region as in the preceding problem. Find the centroid of B. (Hint: by symmetry, $\bar{x} = 0 = \bar{y}$.)

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