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Math 2030, Winter 2005, Final Exam

April 27, 2005

Each problem is worth 10 points unless otherwise indicated.

1. (5) Draw the diagram which shows the relation between (x, y) and (r, θ). Write the formulas for x and y in terms of r and θ, for $dA = dx\,dy$ in terms of r and θ, and for $dV = dx\,dy\,dz$ in terms of r, θ and z.

2. (5) Draw the diagram which shows the relation between (r, z) and (ρ, ϕ). Write the formulas for r and z in terms of ρ and ϕ, for $dA = dr\,dz$ in terms of ρ and ϕ, and for $dV = dx\,dy\,dz$ in terms of ρ, ϕ, and θ.

3. Decompose $(1, 2, 3)$ as the sum of a vector parallel to $(2, 2, 1)$ and a vector perpendicular to $(2, 2, 1)$.

4. Find the point of $2x - y - z = 18$ closest to the point $(1, 1, 1)$.

5. (20) Let $\vec{r}(t) = (t^2, t^3 - t)$.
   (a) Find the velocity $\vec{v} = \vec{r}\,'$.
   (b) Find the speed at time $t = 1$.
   (c) Find the tangent line to $\vec{r}(t)$ at $t = 1$.
   (d) Find the acceleration $\vec{a} = \vec{r}\,''$.
   (e) Decompose $\vec{a}(1)$ into tangential and normal components.
   (f) Compute the curvature κ at that point.

6. Find all first and second partial derivatives of $\sin(xe^y)$.

7. Find the tangent plane to the surface $z = x^2y + y^2$ at the point $(x, y) = (1, 2)$.

8. In which direction does $f(x, y, z) = x^2y + y^2z$ increase most rapidly, at the point $(x, y, z) = (1, 2, 3)$? What is the tangent plane to the level surface of f at this point?

9. Compute $\partial z/\partial x$ and $\partial z/\partial y$ if $e^x + e^y + e^z = 1$.

10. Is the function $f(x, y)$ defined below continuous?

$$f(x,y) = \begin{cases} \dfrac{1 - \sqrt{x+y}}{x+y-1} & x + y \neq 1 \\ 1 & x + y = 1 \end{cases}$$

11. (20) Let L_1 be the line $L_1(t) = (2,0,0) + t(-2,1,0)$ and let L_2 be the line $L_2(s) = (0,2,0) + s(0,1,1)$ in $\mathbf{R}^3$. Find the minimum distance between them, and the points on them closest to one another by:

    (a) using vector methods.

    (b) minimizing an appropriate function of two variables.

    (*Hint: If you do not get the same answer by these two methods, it is probably best to finish the rest of the test before trying to find your error.*)

12. Let B be the region inside the sphere $\rho = 4$ and outside the cylinder $r = 1$. Compute the volume of B.

13. Let R be the region below $y = 4 - x^2$ in the first quadrant. Find the moment of inertia of R about the y-axis.

14. Find and classify the critical points of $f(x,y) = x^2y + y^2 + x^2 + y$.

15. Find the absolute maximum and minimum value of $xy - x$ on the region above $y = x^2$ and below $y = 4$.

16. Compute $\int_C x^2y\,dy$ where C is the parabola $y = x^2$, $0 \le x \le 3$.

17. Compute $\int_C \nabla f \cdot d\vec{r}$ where $f(x,y) = 2x - y^2$ and C is a curve which starts at $(2,1)$ and ends at $(1,5)$.

18. Let C_1 and C_2 be circles of radius 1 centered at $(1,0)$ and $(-1,0)$ respectively. Let C_3 be the circle of radius 10 centered at the origin and let R be the region inside C_3 and outside both C_1 and C_2. If $Q_x - P_y = 1$ in R, $\int_{C_1} P\,dx + Q\,dy = 8\pi$, and $\int_{C_2} P\,dx + Q\,dy = 10\pi$, what is $\int_{C_3} P\,dx + Q\,dy$?

19. Let $\mathcal{S}$ be the part of the surface $z = 2x + 5y + 7$ which lies above the rectangle $-1 \le x \le 2$, $0 \le y \le 2$. Find the surface area of $\mathcal{S}$ and the surface integral $\iint_{\mathcal{S}} z\,\vec{k} \cdot d\vec{S}$.

——— The End ———