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Math 2030, Winter 2004, Test 3
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1. Draw the diagram which shows the relation between (x, y) and (r, θ). Write the formulas for x and y in terms of r and θ, and for $dA = dx\,dy$ in terms of r and θ.

2. Draw the diagram which shows the relation between (r, z) and (ρ, ϕ). Write the formulas for r and z in terms of ρ and ϕ, and for $dA = dr\,dz$ in terms of ρ and ϕ.

3. Suppose that $x \leq f(x, y) \leq x + y$ for all (x, y). What can you say about $\iint_R f\,dA$ if R is the rectangle $[0, 2] \times [0, 3]$?

4. Find the area and centroid of the triangle with edges $y = x$, $x = 2y$, and $y = 2$.

5. Let R be the eighth of the disk $x^2 + y^2 \leq 1$ which lies above the x-axis and below the line $y = x$. The area of R is thus $\pi/8$. Find $\bar{x}$, the x-coordinate of the centroid of R.

6. Let $(x, y) = F(u, v) = (u + v^2, v - u^2)$.

   (a) Find the Jacobian of F,
   $$\frac{\partial(x, y)}{\partial(u, v)}$$

   (b) If R is the rectangle $[0, 1] \times [0, 1]$ in the uv-plane, find the area of its image, $F(R)$.

7. Let R be the wedge shaped slice of the cylinder $0 \leq r \leq 1$ which lies between the planes $z = 0$ and $z = 1 - x$. Find the volume of R.

8. Let R be the region inside the hyperboloid $r^2 = 1 + z^2$, between the planes $z = 1$ and $z = -1$. Find the moment of inertia of R about the z-axis.

9. Find the volume inside $\rho = \cos(\phi)$, $0 \leq \phi \leq \pi/2$.

10. Let B be the eighth of a solid ball which lies in the first octant (so that $0 \leq \theta \leq \pi/2$, and $0 \leq \phi \leq \pi/2$). The volume of B is thus $(\frac{4\pi}{3})/8 = \pi/6$. Find $\bar{z}$, the z-coordinate of the centroid of B.

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