Math 2030, Winter 2011, Test 2

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March 4, 2011

1. (10) Find all first and second partial derivatives of $x^2 \cos(y)$.

2. Let $f(x, y) = \dfrac{xy}{x+y}$.

   (a) (10) Find the tangent plane to $z = f(x, y)$ at $(x, y) = (1, 2)$.

   (b) (5) In what direction does $f(x, y)$ *increase* most rapidly at $(x, y) = (1, 2)$.

   (c) (5) Find a direction in which $f(x, y)$ neither increases nor decreases at $(1, 2)$.

3. (a) (10) Find the tangent plane to the level surface $xy^2 + yz^3 = 6$ at $(x, y, z) = (1, 2, 1)$.

   (b) (5) Intersecting this level surface with the plane $x - y + z = 0$ gives a curve passing through $(1, 2, 1)$. Find a vector tangent to this curve at $(1, 2, 1)$.

4. (10) Suppose that $f(x, y) = xy^2$, $\partial x/\partial s = 1$, $\partial x/\partial t = 2$, $\partial y/\partial s = 3$, and $\partial y/\partial t = 4$. Find $\partial f/\partial s$ and $\partial f/\partial t$.

5. Suppose $C(x, y, z) = xy^2z^3$.

   (a) (5) Find the differential dC.

   (b) (5) If we measure x, y, and z with relative error at most .01, what is the relative error in our knowledge of C?

6. (5) Find $\partial z/\partial x$ and $\partial z/\partial y$ if $xyz + xy + yz = 3$.

7. (10) Find and classify the critical points of $4y - y^2 + x^2y - 4x^2$.

8. (10) Find the absolute maximum and minimum values of $x^2y + y^3$ on the triangle with vertices $(-1, -1)$, $(-1, 2)$, and $(2, -1)$.

9. (10) Find the maximum and minimum values of $C = xy + 2xz + 3yz$ on the surface $xyz = 6$.

——— The End ———