Solutions

Math 2030, Winter 2011, Quiz 4
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No calculators needed or allowed.

Let $\vec{r}(t) = (t, 2t, 2t^2)$.

1. Compute the velocity $\vec{v}(t) = \vec{r}\,'(t)$
2. Compute the speed ds/dt.
3. Compute the acceleration $\vec{a}(t) = \vec{r}\,''(t)$
4. Compute the tangent line at $\vec{r}(1)$.
5. Decompose the acceleration $\vec{a}(1)$ into components parallel to and perpendicular to the velocity $\vec{v}(1)$.
6. Show that the curve lies in the intersection of the surface $z = xy$ and the plane $2x - y = 0$.

Answers:

1. $\vec{v}(t) = (1, 2, 4t)$
2. $ds/dt = \sqrt{5 + 16t^2}$.
3. $\vec{a}(t) = (0, 0, 4)$
4. The tangent line at $\vec{r}(1) = (1, 2, 2)$ has direction $\vec{v}(1) = (1, 2, 4)$, so the equation of the line is $(1, 2, 2) + t(1, 2, 4) = (1 + t, 2 + 2t, 2 + 4t)$.
5. The component of the acceleration $\vec{a}(1)$ in the direction of $\vec{v}(1)$ is
$$\frac{\vec{a}(1)\cdot\vec{v}(1)}{\vec{v}(1)\cdot\vec{v}(1)}\vec{v}(1) = \frac{(0,0,4)\cdot(1,2,4)}{(1,2,4)\cdot(1,2,4)}(1,2,4) = \frac{16}{21}(1,2,4)$$
and the perpendicular component is
$$(0,0,4) - \frac{16}{21}(1,2,4) = \frac{1}{21}(-16,-32,20).$$
6. It suffices to show that the curve lies in each surface. Since $(x, y, z) = (t, 2t, 2t^2)$ on the curve, we have $xy = t(2t) = 2t^2 = z$ and $2x - y = 2t - 2t = 0$.