Name:_____

Math 2030, Winter 2011, Quiz 8
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No calculators needed or allowed.

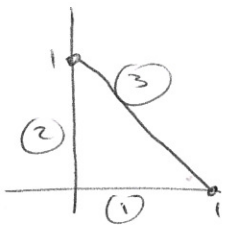
Find the maximum and minimum values of $x^2 - xy$ on the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

Let $f(x,y) = x^2 - xy$. Then $f_x = 2x - y$

and $f_y = -x$

are both true simultaneously.

So critical points occur where $x = 0$ and $2x = y$. Thus $(0,0)$ is the only critical point. $f(0,0) = 0$. (0,0) 2



Boundary curves:

(1) $y = 0$ $\quad 0 \le x \le 1$

$f(x,0) = x^2$. Min = 0 at 0, Max = 1 at $x = 1$. (0,0) (1,0) 2

(2) $x = 0$, $0 \le y \le 1$.

$f(0,y) = 0$ $\quad$ Min = Max = 0 everywhere. (0,y) 2

(3) $x + y = 1$, $0 \le x \le 1$, so $y = 1 - x$. Then

$f(x,y) = f(x, 1-x) = x^2 - x(1-x) = 2x^2 - 1$. 2

$f' = 4x - 1$ so C.P. @ $x = 1/4$. Then $y = 1 - x = 3/4$.

$f\left(\frac{1}{4}, \frac{3}{4}\right) = \frac{1}{16} - \frac{3}{16} = \frac{-1}{8}$. $\left(\frac{1}{4}, \frac{3}{4}\right)$

Corners: $(0,0)$, $(0,1)$, $(1,0)$ have all already been considered. 2

Max = 1 at $(1,0)$, $\quad$ min = $-1/8$ at $(1/4, 3/4)$.