

Name: _____

Math 2030, Winter 2011, Quiz 7
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No calculators needed or allowed.

Find and classify the critical points of $f(x, y) = x^4 - xy + 2y^2$.
Organize your work clearly, showing

- the derivatives,
- the critical points,
- the second derivatives,
- the discriminant at each critical point, and
- the type of each critical point.

Answers:

$$f_x = 4x^3 - y$$

$$f_y = -x + 4y$$

Critical points: $x = 4y$ and $y = 4x^3$, so $x = 4(4x^3) = 16x^3$.

$$\text{Then } x(16x^2 - 1) = 0 \text{ so } x = 0, y = 0$$

or $x = \pm \frac{1}{4}$, $y = \pm \frac{1}{16}$

$(0, 0)$
 $(\frac{1}{4}, \frac{1}{16})$
 $(-\frac{1}{4}, -\frac{1}{16})$

	$(0, 0)$	$(\frac{1}{4}, \frac{1}{16})$	$(-\frac{1}{4}, -\frac{1}{16})$
$f_{xx} = 12x^2$	0	$\frac{3}{4}$	$\left\{ \begin{array}{l} \text{Same as } (\frac{1}{4}, \frac{1}{16}) \\ \text{So} \end{array} \right.$
$f_{xy} = -1$	-1	-1	
$f_{yy} = 4$	4	4	$\left\{ \begin{array}{l} \text{Min} \\ \text{Min} \end{array} \right.$
D	$0 - (-1)^2 < 0$ saddle	$4(\frac{3}{4}) - (-1)^2 = 3 > 0$ $f_{xx} > 0$ min	