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Math 2030, Winter 2011, Quiz 2
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No calculators needed or allowed.

Let $P = (2,1,2)$ and $Q = (1,1,1)$. Consider the plane

$$(1,2,1)\cdot(x,y,z) = 4 \qquad (*)$$

1. Show that Q is in the plane but P is not.
2. Compute $\text{proj}_{\vec{n}}\vec{PQ}$, the component of $\vec{PQ}$ perpendicular to the plane.
3. Write $\vec{PQ}$ as the sum of a vector parallel to the plane and a vector perpendicular to the plane.
4. Find the distance from P to the plane.
5. Find the point in the plane closest to P.

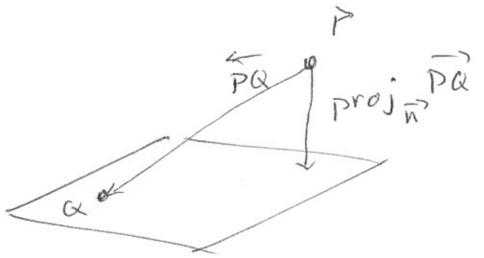
Answers: $\vec{n} = (1,2,1)$ is normal to the plane.

1. $\vec{n}\cdot Q = (1,2,1)\cdot(1,1,1) = 1+2+1 = 4$, so Q solves $(*)$
$\vec{n}\cdot P = (1,2,1)\cdot(2,1,2) = 2+2+2 = 6$, so P doesn't solve $(*)$.

2. $\text{proj}_{\vec{n}}\vec{PQ} = \text{proj}_{(1,2,1)}\overrightarrow{(-1,0,-1)} = \dfrac{-1+0-1}{1+4+1}(1,2,1) = \frac{-1}{3}(1,2,1)$

3. $(-1,0,-1) = \underbrace{-\frac{1}{3}(1,2,1)}_{\perp \text{ to plane}} + \underbrace{\left(\frac{-2}{3},\frac{2}{3},\frac{-2}{3}\right)}_{\text{parallel to plane}}$

4. distance to plane $= |\text{proj}_{\vec{n}}\vec{PQ}| = \frac{1}{3}\sqrt{1+4+1} = \frac{\sqrt{6}}{3}$

5. 

$P + \text{proj}_{\vec{n}}\vec{PQ} = (2,1,2) - \frac{1}{3}(1,2,1)$
$= \left(\frac{5}{3},\frac{1}{3},\frac{5}{3}\right)$

Check: $(1,2,1)\cdot\left(\frac{5}{3},\frac{1}{3},\frac{5}{3}\right) = \frac{5+2+5}{3} = \frac{12}{3} = 4$

so this point is in the plane.