Name:_____

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One the the following vector fields is conservative:

$$(2xy, x^2) \qquad \text{and} \qquad (2xy, y^2)$$

1. Determine which one it is and call it $\mathbf{F}$.
2. Find its potential function.
3. If C is a curve starting at $(1,2)$ and ending at $(5,4)$, what is the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$?

1.

(P,Q)	$(2xy, x^2)$	$(2xy, y^2)$
$Q_x \stackrel{?}{=} P_y$	$2x = 2x$	$0 \neq 2x$
conservative?	possible	No
	F	

2. If $F = \nabla f$ then $f_x = 2xy$ and $f_y = x^2$

Then $f = \int f_x \, dx + g(y)$
$= \int 2xy \, dx + g(y)$
$= x^2y + g(y)$

→ so $f_y = x^2 + g'(y)$. Thus $g'(y) = 0$ and we may let $g = 0$.

Check: $\nabla(x^2y) = (2xy, x^2)$. This shows $(2xy, x^2)$ is conservative

potential function $f = x^2y$

3. $$\int_C \vec{F} \cdot d\vec{r} = x^2y \Big|_{(1,2)}^{(5,4)} = 25 \cdot 4 - 1 \cdot 2 = 98.$$