Name:______________________________

Math 2030, Winter 2011, Quiz 11
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Let R be the solid region above the xy-plane and below the paraboloid $z = 16 - r^2$. Assuming constant density 1, find

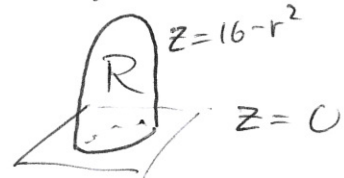
1. the volume of R,
2. the moment of inertia about the z-axis.

Hint: Write powers in exponential form. Don't multiply them out. For example,

$$4^5 - \frac{4^6}{6} = 4^5(1 - \frac{4}{6}) = \cdots$$

is easier to work with than it would be if you had replaced 4^5 by 1024 and 4^6 by 4096.
Extra credit: Find the centroid of R.

(2)



$z=16-r^2$ intersects $z=0$ when $r^2=16$ so $0 \le r \le 4$.

The Region

1. $$\int_0^{2\pi}\int_0^4\int_0^{16-r^2} r\,dz\,dr\,d\theta$$

$$= 2\pi \int_0^4 r(16-r^2)\,dr$$

$$= 2\pi\left[8r^2 - \frac{r^4}{4}\Big|_0^4\right] = 2\pi\left[8\cdot 4^2 - \frac{4^4}{4}\right]$$

$$= 2\pi\left[2\cdot 4^3 - 4^3\right] = 2\pi\, 4^3 = \boxed{2^7\pi}$$

(4)

2. $$I_{z\text{-axis}} = \iiint_R r^2\,dV = \int_0^{2\pi}\int_0^4\int_0^{16-r^2} r^3\,dz\,dr\,d\theta = 2\pi\int_0^4 r^3(16-r^2)\,dr$$

(4)

$$= 2\pi\left[4r^4 - \frac{r^6}{6}\Big|_0^4\right] = 2\pi\left[4^5 - \frac{4^6}{6}\right] = 2\pi\cdot 4^5\left(1-\frac{4}{6}\right) = \boxed{\frac{2^{11}\pi}{3}}$$

E.C. $\bar{x} = \bar{y} = 0$ by symmetry about the z-axis (2)

$$\iiint_R z\,dV = \int_0^{2\pi}\int_0^4\int_0^{16-r^2} rz\,dz\,dr\,d\theta = \frac{2\pi}{2}\int_0^4 r(16-r^2)^2\,dr$$

Subst. $u = 16 - r^2$
$u(0) = 16$
$u(4) = 0$
$du = -2r\,dr$
$-\frac{1}{2}du = r\,dr$

$$= \pi\int_{16}^0 u^2\left(-\tfrac{1}{2}du\right) = \frac{\pi}{2}\int_0^{16} u^2\,du = \frac{\pi}{2}\,\frac{16^3}{3} = \frac{2^{11}\pi}{3}$$

$$\bar{z} = \frac{2^{11}\pi}{3} \Big/ 2^7\pi = \frac{2^4}{3} = \frac{16}{3}$$

(3)