

Name: _____

Math 2030, Winter 2011, Quiz 10

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R. Bruner

1. If $x = u + v$ and $y = uv$, calculate $\partial(x, y)/\partial(u, v)$.
2. Compute the area inside the cardioid $r = 2 + \cos(\theta)$.

1.
$$\frac{\partial(x, y)}{\partial(u, v)} = \det \begin{pmatrix} 1 & 1 \\ v & u \end{pmatrix} = u - v$$

2.
$$\int_0^{2\pi} \int_0^{2+\cos\theta} r \, dr \, d\theta = \int_0^{2\pi} \frac{1}{2} (2+\cos\theta)^2 \, d\theta$$

$$= \int_0^{2\pi} 2 + 2\cos\theta + \frac{1}{2}\cos^2\theta \, d\theta$$

$$= 2\theta + 2\sin\theta + \frac{\theta + \sin\theta\cos\theta}{4} \Big|_0^{2\pi}$$

$$= 4\pi + 0 + \frac{2\pi + 0}{4} - (0 + 0 + 0)$$

$$= \boxed{4\pi + \frac{\pi}{2}} = \frac{9}{2}\pi$$