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Math 2030, Winter 2005, Quiz 13
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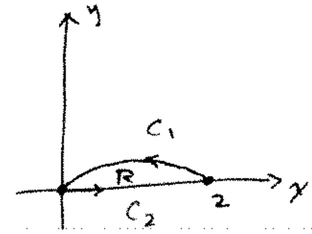
Let R be the region in the first quadrant with lower boundary the x-axis and upper boundary formed by the curve $x = 1 - t^3$, $y = 1 - t^2$ for $-1 \le t \le 1$. Use Green's Theorem to compute the area and centroid of R by computing the appropriate line integrals.

Hint: Integrals along the lower boundary of the form $\int$ something dy will be zero since y is constant along the x-axis.



$$\text{Area}(R) = \iint_R 1\, dA = \oint_C x\,dy = \oint_{C_1} x\,dy \quad \text{by } (*)$$

$$= \int_{-1}^{1} (1-t^3)(-2t)\,dt = 2\int_{-1}^{1} t^4 - t\,dt = 4\int_0^1 t^4\,dt = \boxed{\frac{4}{5}}$$

$$(*) \quad \oint_{C_2} \text{something } dy = 0$$

C_1: $x = 1 - t^3$
$y = 1 - t^2$
$dx = -3t^2\,dt$
$dy = -2t\,dt$
$-1 \le t \le 1$

$$\bar{x}: \quad \iint_R x\,dA = \oint_C \tfrac{1}{2}x^2\,dy = \oint_{C_1} \tfrac{1}{2}x^2\,dy \quad \text{by } (*)$$

$$= \frac{1}{2}\int_{-1}^{1} (1-t^3)^2(-2t)\,dt = -\int_{-1}^{1} t - 2t^4 + t^7\,dt$$

$$= 2\int_{-1}^{1} t^4\,dt = 4/5 \text{ as above, so } \boxed{\bar{x} = 1}$$

$$\bar{y}: \quad \iint_R y\,dA = \oint_C xy\,dy = \oint_{C_1} xy\,dy \quad \text{by } (*)$$

$$= \int_{-1}^{1} (1-t^3)(1-t^2)(-2t)\,dt = -2\int_{-1}^{1} t - t^3 - t^4 + t^6\,dt = -2\int_{-1}^{1} t^6 - t^4\,dt = -4\left[\frac{1}{7} - \frac{1}{5}\right]$$

$$= 4\,\frac{2}{35} = \frac{8}{35} \quad \text{so} \quad \bar{y} = \frac{8/35}{4/5} = \boxed{\frac{2}{7}}$$