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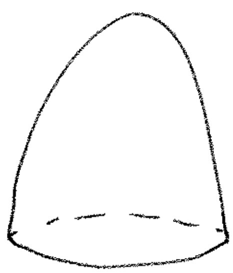
Let R be the solid region above the xy-plane and below the paraboloid $z = 16 - r^2$. Find the volume and moment of inertia about the z-axis, assuming constant density 1.



$$0 \le \theta \le 2\pi$$
$$0 \le r \le 4$$
$$0 \le z \le 16 - r^2$$

$$\text{Vol} = \int_0^{2\pi}\int_0^4\int_0^{16-r^2} r\,dz\,dr\,d\theta = 2\pi\int_0^4 r(16-r^2)\,dr$$

$$\left(\begin{array}{l|l} u = 16-r^2 & r=0 \Rightarrow u=16 \\ du = -2r\,dr & r=4 \Rightarrow u=0 \\ -\tfrac{1}{2}du = r\,dr & \end{array}\right)$$

$$= \frac{2\pi}{2}\int_0^{16} u\,du = \frac{\pi}{2}u^2\Big|_0^{16} = \frac{256\pi}{2} = \boxed{128\pi}$$

$$I_z = \int_0^{2\pi}\int_0^4\int_0^{16-r^2} r^2 r\,dz\,dr\,d\theta = 2\pi\int_0^4 r^3(16-r^2)\,dr$$

$$= \frac{2\pi}{2}\int_0^{16} u(16-u)\,du = \pi\int_0^{16} 16u - u^2\,du = \pi\left[8u^2 - \frac{u^3}{3}\Big|_0^{16}\right]$$

$$= \pi\left[8\cdot 16^2 - \frac{16^3}{3}\right] = 8\cdot 16^2\pi\left[1-\frac{2}{3}\right] = \frac{8\cdot 16^2\pi}{3} = \boxed{\frac{2^{11}\pi}{3}}$$

So $I_z = \text{Vol} * \dfrac{2^4}{3}$ since $\text{Vol} = 2^7\pi$