R. Bruner
Math 2030, Winter 2016, Final Exam
April 27, 2016

Each problem is worth 10 points unless otherwise indicated.

1. Draw the diagram which shows the relation between (x, y) and (r, θ).
   (a) Write the formula for x and y in terms of r and θ.
   (b) Write the formula for $dx\,dy$ in terms of r, θ, dr, and $d\theta$.
   (c) Write the formula for $dx\,dy\,dz$ in terms of r, θ, z, dr, $d\theta$ and dz.
2. Draw the diagram which shows the relation between (r, z) and (ρ, ϕ).
   (a) Write the formula for r and z in terms of ρ and ϕ.
   (b) Write the formula for $dr\,dz$ in terms of ρ, ϕ, $d\rho$, and $d\phi$.
   (c) Write the formula for $dx\,dy\,dz$ in terms of ρ, ϕ, θ, $d\rho$, $d\phi$, and $d\theta$.
3. Decompose $(1, -11, 1)$ as the sum of a vector parallel to $(1, 3, -1)$ and a vector perpendicular to $(1, 3, -1)$.
4. Find the point of $x + 3y - 2z = -9$ closest to the point $(7, 18, -7)$.
5. Find a parametric equation for the line of intersection of the planes $x + y - z = 0$ and $2x - y - z = 1$.
6. (20) Let $\mathbf{r}(t) = (t^3 - t, t^2 - 1)$.
   (a) Find the velocity $\mathbf{v} = \mathbf{r}'$.
   (b) Find the speed at time $t = 1$.
   (c) Find the tangent line to $\mathbf{r}(t)$ at $t = 1$.
   (d) Find the acceleration $\mathbf{a} = \mathbf{r}''$.
   (e) Decompose $\mathbf{a}(1)$ into tangential and normal components.
   (f) Compute the curvature κ at that point.
7. Find all first and second partial derivatives of $x^3 \sin(y^2 + 1)$.
8. Find the tangent plane to the surface $z = x^3 - xy^2$ at the point $(x, y) = (1, 2)$.

——— Over ———

9. (15) Let $f(x, y, z) = x^3 + xyz + z^3$.

   (a) In which direction does f increase most rapidly at the point $(x, y, z) = (1, 2, 3)$?

   (b) What is the tangent plane to the level surface of f at $(1, 2, 3)$?

   (c) Compute $\partial z/\partial x$ and $\partial z/\partial y$ if z is determined by this level surface, i.e., by the condition $x^3 + xyz + z^3 = 34$.

10. (15) Let R be the region between $y = 3x - x^2$ and the x-axis. Find the area and centroid of R.

11. Find and classify the critical points of $f(x, y) = x^3 + xy^2 - 3x$.

12. Find the absolute maximum and minimum value of $x^2 - xy$ on the region below $y = 3$ and above $y = x^2 - 1$.

13. (15) Let B be the region given by $0 \leq z \leq r^2 - r^3$ and $0 \leq r \leq 1$ in cylindrical coordinates.

   (a) Find the volume of B.

   (b) Find the centroid of B. (Hint: you may use symmetry to note that the x and y coordinates are obvious, but don't forget to say what they are.)

14. Compute $\int_C x\,dy - y\,dx$ where C is the parabola $y = x^2$, $0 \leq x \leq 3$.

15. Compute $\int_C \nabla f \cdot d\vec{r}$ where $f(x, y) = xy$ and C is a curve which starts at $(2, 1)$ and ends at $(1, 5)$.

16. Let R be the rectangle $0 \leq x \leq 3,\ \ 0 \leq y \leq 2$. If $Q_x - P_y = 5$, evaluate $\int_{\partial R} P\,dx + Q\,dy$.

17. (15) Let $\mathcal{S}$ be the part of the surface $z = x + y + 1$ which lies inside the cylinder $x^2 + y^2 = 4$. Let $\mathbf{F}(x, y, z) = (x, x, 0)$.

   (a) Find the surface area of $\mathcal{S}$.

   (b) Compute $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \cdot d\mathbf{S}$.

   (c) Compute $\int_{\partial \mathcal{S}} \mathbf{F} \cdot d\mathbf{r}$.

——— The End ———