

R. Bruner
Math 2030, Winter 2017, Test 3
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1. Polar and cylindrical coordinates: Draw the diagram which shows the relation between (x, y) and (r, θ) . Write the formulas for
 - (a) x and y in terms of r and θ ,
 - (b) for $dA = dx dy$ in terms of r, θ, dr and $d\theta$,
 - (c) for $dV = dx dy dz$ in terms of $r, \theta, z, dr, d\theta$ and dz .
2. Spherical coordinates: Draw the diagram which shows the relation between (r, z) and (ρ, ϕ) . Write the formulas for
 - (a) r and z in terms of ρ and ϕ ,
 - (b) for $dA = dr dz$ in terms of $\rho, \phi, d\rho$ and $d\phi$,
 - (c) for $dV = dx dy dz$ in terms of $\rho, \theta, \phi, d\rho, d\theta$ and $d\phi$.
3. Find the volume of the region below $z = 2x + 4y$ and above the triangle in the xy -plane bounded by the positive x -axis, the positive y -axis, and the line $2x + y = 4$.
4. Let C be the cone whose base lies in the xy -plane and whose top surface is given by $z = 4 - r$ in cylindrical coordinates. Find the volume of C .
5. Let S be the ‘slice’ of the sphere $\rho = 1$ which lies between the xy -plane and the angle $\phi = \pi/4$. Find the volume of S .
6. Let $(x, y) = (u + v^2, v - u^2)$ be a transformation from the uv -plane to the xy -plane.
 - (a) Find the Jacobian $\partial(x, y)/\partial(u, v)$.
 - (b) Compute the area of the region in the xy -plane which is the image under this transformation of the unit square $[0, 1] \times [0, 1]$ in the uv -plane.
7. Convert from spherical to cylindrical and then to rectangular, coordinates:
 - (a) $\rho = 2$
 - (b) $\phi = \pi/4$
8. Convert from cylindrical to rectangular coordinates:
 - (a) $r = 3$
 - (b) $\theta = \pi/4$
9. (Extra credit) Describe the region in the xy -plane whose area you calculated in problem 6.

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