1. $f(x,y) = \sin(x^2+y)$

$f_x = 2x\cos(x^2+y)$ $\qquad f_{xx} = 2\cos(x^2+y) - 4x^2\sin(x^2+y)$

$f_y = \cos(x^2+y)$ $\qquad f_{xy} = -2x\sin(x^2+y)$

$f_{yy} = -\sin(x^2+y)$

2. $f(x,y,z) = x^2yz + y^2z^3$

(a) $\boxed{\nabla f = (2xyz,\ x^2z + 2yz^3,\ x^2y + 3y^2z^2)}$

(b) $\nabla f(1,1,1) = (2,3,4)$ so f decreases most rapidly in the direction of $\boxed{-(2,3,4)}$.

(c) $\boxed{(3,-2,0),\ (4,0,-2) \text{ and } (0,4,-3) \text{ are obvious choices.}}$

3. $f(x,y) = x^3 - xy$

(a) $\nabla f = (3x^2-y,\ -x)$ so $\nabla f(2,3) = (12-3, -2) = (9,-2)$

and $f(2,3) = 8-6 = 2$ so the tangent plane is

$$\boxed{z-2 = 9(x-2) - 2(y-3)}$$

(b) $\boxed{df = (3x^2-y)dx - x\,dy}$

(c) $f(2.1, 3.2) \simeq f + df = 2 + 9(.1) - 2(.2)$

$= 2 + .9 - .4 = \boxed{2.5}$

4. (a) $\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}$ give $\dfrac{xy^2}{x^2+y^2} = \dfrac{r^3\cos\theta\sin^2\theta}{r^2} = r\cos\theta\sin^2\theta \to 0$

as $(x,y)\to(0,0)$. $\boxed{\lim = 0.}$

(b) $$\frac{x^2-y^2}{x^4+y^4} = \frac{r^2(\cos^2\theta - \sin^2\theta)}{r^4(\cos^4\theta+\sin^4\theta)} = \frac{1}{r^2}\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^4\theta+\sin^4\theta}\right)$$

$\frac{1}{r^2} \to \infty$ while $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^4\theta+\sin^4\theta} = \begin{cases} 1 & \theta = 0 \\ 0 & \theta = \pi/4 \end{cases}$

so the $\boxed{\text{limit Does Not Exist.}}$

5. $xy^2z + x^2y - yz^2 = 1 \quad (*)$

(a) $\nabla = (y^2z + 2xy,\ 2xyz + x^2 - z^2,\ xy^2 - 2yz)$

$= (3, 2, -1)$ at $(x, y, z) = (1, 1, 1)$

The tangent plane at $(1,1,1)$ is therefore

$$\boxed{3(x-1) + 2(y-1) - (z-1) = 0}$$

(b) Normals to the tangent plane and $x+y-z=0$ are perpendicular to the tangent line, so its direction is

$$\begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = (-2+1,\ -(-3+1),\ 3-2) = (-1, 2, 1)$$

The $\boxed{\text{tangent line is } \vec{r}(t) = (1,1,1) + t(-1,2,1).}$

(c) Applying $\frac{\partial}{\partial x}$ to $(*)$ we get

$$y^2z + xy^2\frac{\partial z}{\partial x} + 2xy - 2yz\frac{\partial z}{\partial x} = 0$$

so $$\boxed{\frac{\partial z}{\partial x} = -\frac{y^2z + 2xy}{xy^2 - 2yz} = -\frac{yz + 2x}{xy - 2z}.}$$

6. $f(x,y) = y^2 - 2x^3y + 3x^2$

$f_x = -6x^2y + 6x$
$\quad = 6x(1-xy)$

$f_y = 2y - 2x^3$
$\quad = 2(y - x^3)$

C.P.

$f_x = 0$
$\Downarrow$

	$x=0$ OR	$xy=1$
$f_y=0 \Rightarrow y=x^3$	$(0,0)$	$x \cdot x^3 = 1 \Rightarrow x = \pm 1$ $(1,1)$ or $(-1,-1)$

	$(0,0)$	$(1,1)$	$(-1,-1)$
$f_{xx} = -12xy + 6$	6	-6	-6
$f_{xy} = -6x^2$	0	-6	-6
$f_{yy} = 2$	2	2	2
$D = f_{xx}f_{yy} - f_{xy}^2$	$12 > 0$ $f_{xx} > 0$ Min	$-12-36 < 0$ saddle	$-12-36 < 0$ saddle

7. $L = x + 2y - \lambda(x^2+y^2-1)$

$L_x = 1 - 2x\lambda$

$L_y = 2 - 2y\lambda$

C.P. $\Rightarrow \lambda = \frac{1}{2x} = \frac{2}{2y} = \frac{1}{y}$

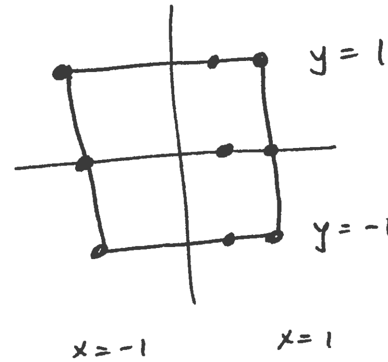
$\Rightarrow y = 2x$

$\Rightarrow x^2 + (2x)^2 = 1$ or $5x^2 = 1$ so $x = \frac{\pm 1}{\sqrt{5}}$

Critical points $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $\left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$

$$\boxed{\begin{aligned} \text{Max} &= \frac{1}{\sqrt{5}} + 2\left(\frac{2}{\sqrt{5}}\right) = \sqrt{5} \\ \text{Min} &= \frac{-1}{\sqrt{5}} + 2\left(\frac{-2}{\sqrt{5}}\right) = -\sqrt{5} \end{aligned}}$$

$\boxed{8.}$ Max/min of $x^2 - x + y^2$ on $|x| \le 1, |y| \le 1$



Interior: $\left.\begin{array}{l} f_x' = 2x - 1 \\ f_y = 2y \end{array}\right]$ C.P. $\Rightarrow$ $x = 1/2$, $y = 0$

$(\frac{1}{2}, 0)$

Boundary: $x = 1$: $f = y^2$, $f' = 2y$ C.P. $y = 0$ $(1, 0)$

$x = -1$ $f = 2 + y^2$, $f' = 2y$ C.P. $y = 0$ $(-1, 0)$

$y = 1$ $f = x^2 - x + 1$, $f' = 2x - 1$ C.P. $x = 1/2$ $(\frac{1}{2}, 1)$

$y = -1$ $f = x^2 - x + 1$ again $(\frac{1}{2}, -1)$

Values:

	(x,y)	$f(x,y)$	
Interior	$(\frac{1}{2}, 0)$	$\frac{1}{4} - \frac{1}{2} + 0 = -1/4$	MIN at $(\frac{1}{2}, 0)$
Boundary	$(1, 0)$	$1 - 1 + 0 = 0$	
	$(-1, 0)$	$1 + 1 + 0 = 2$	
	$(\frac{1}{2}, 1)$ or $(\frac{1}{2}, -1)$	$\frac{1}{4} - \frac{1}{2} + 1 = 3/4$	
Corners	$(-1, -1)$	$1 + 1 + 1 = 3$	MAX at $(-1, \pm 1)$
	$(-1, 1)$	$1 + 1 + 1 = 3$	
	$(1, -1)$	$1 - 1 + 1 = 1$	
	$(1, 1)$	$1 - 1 + 1 = 1$	