

**Math 2030, Fall 2017, Test 2**  
**18 October 2017**  
**R. Bruner**

1. (10) Find all first and second partial derivatives of  $\sin(x^2 + y)$ .
2. Let  $f(x, y) = x^3 - xy$ .
  - (a) (10) Find the tangent plane to  $z = f(x, y)$  at  $(x, y) = (2, 3)$ .
  - (b) (5) Compute the differential  $df$ .
  - (c) (5) Estimate  $f(2.1, 3.2)$  using either (a) or (b).
3. Let  $f(x, y, z) = x^2yz + y^2z^3$ .
  - (a) (5) Compute  $\nabla f$ .
  - (b) (5) In what direction does  $f(x, y)$  decrease most rapidly at  $(x, y) = (1, 1, 1)$ .
  - (c) (5) Find two distinct directions in which  $f(x, y)$  neither increases nor decreases at  $(1, 1, 1)$ .
4. (10) Compute or show these do not exist:
  - (a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$
  - (b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^4 + y^4}$
5. (a) (10) Find the tangent plane to the level surface  $xy^2z + x^2y - yz^2 = 1$  at  $(x, y, z) = (1, 1, 1)$ .
  - (b) (5) Intersecting this level surface with the plane  $x + y - z = 1$  gives a curve passing through  $(1, 1, 1)$ . Find its tangent line there.
  - (c) (5) If we use this level surface to define  $z$  as a function of  $(x, y)$ , find the partial derivative  $\partial z / \partial x$ .
6. (15) Find and classify the critical points of  $y^2 - 2x^3y + 3x^2$ .
7. (10) Find the maximum and minimum values of  $x + 2y$  on the circle  $x^2 + y^2 = 1$ .
8. (10) Find the absolute maximum and minimum values of  $x^2 - x + y^2$  on the square  $|x| \leq 1$  and  $|y| \leq 1$ .

————— The End —————