Name:______________________________

Math 2030, Fall 2017, Quiz 9
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No calculators needed or allowed. In case you have temporarily forgotten, $3^5 = 243$.

Let $R = [0,4] \times [0,2]$. Evaluate the integral

$$\iint_R x\sqrt{xy+1}\,dA.$$

Hint: the integral $\int x\sqrt{xy+1}\,dy$ can be done using the substitution $u = xy+1$; recall that x is treated as a constant in $\int \cdots dy$, so that $du = xdy$ when doing this substitution.

First, the substitution $u = xy+1$, x constant, gives $du = xdy$

so that $\int_0^2 x\sqrt{xy+1}\,dy = \int_1^{2x+1} \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_1^{2x+1} = \frac{2}{3}\left[(2x+1)^{3/2} - 1\right]$

Thus

$$\int_0^4 \int_0^2 x\sqrt{xy+1}\,dy\,dx = \frac{2}{3}\int_0^4 (2x+1)^{3/2} - 1\,dx$$

$$= \frac{2}{3}\left[\frac{1}{2}\cdot\frac{2}{5}(2x+1)^{5/2} - x\right]\Bigg|_0^4 \quad \boxed{A}$$

$$= \frac{2}{3}\left[\frac{9^{5/2}}{5} - 4 - \left(\frac{1}{5} - 0\right)\right]$$

$$= \frac{2}{3}\left[\frac{243-1}{5} - 4\right]$$

$$= \frac{2}{3}\left[\frac{222}{5}\right] = \frac{444}{15}$$

$$\text{OR} = \frac{2(74)}{5} = \frac{148}{5}$$

check

$$\left(\frac{1}{5}(2x+1)^{5/2} - x\right)'$$

$$= \frac{1}{5}\cdot\frac{5}{2}(2x+1)^{3/2}\cdot 2 - 1$$

$$= (2x+1)^{3/2} - 1$$

Why can't I get rid of the -1; just absorb it into the +C that occurs in antiderivatives?

$\boxed{A}$ $\int (2x+1)^{3/2}\,dx = \frac{1}{2}\int u^{3/2}\,du = \frac{1}{2}\cdot\frac{2}{5}u^{5/2} = \frac{1}{5}u^{5/2}$ *+C, of course*

$u = 2x+1$
$du = 2\,dx$