Name:________________________________

Math 2030, Fall 2017, Quiz 8
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No calculators needed or allowed.

1. Find and classify the critical points of $f(x,y) = x^3 - x^3y - xy + xy^2$.

2. Find the maximum and minimum values of $3x + 4y$ on the curve $x^2 + xy + y^2 = 1$.

1. $f = x^3 - x^3y - xy + xy^2$

$f_x = 3x^2 - 3x^2y - y + y^2$
$= (3x^2 - y)(1-y)$

$f_y = -x^3 - x + 2xy$
$= -x(x^2+1-2y)$

$f_x = 0 \iff y = 1$ OR $y = 3x^2$

$f_y = 0 \iff x = 0$ OR $y = \frac{x^2+1}{2}$

	$f_x = 0$: $y = 1$ OR	$y = 3x^2$
$f_y = 0$: $x = 0$ OR	$(0,1)$	$(0,0)$
$y = \frac{x^2+1}{2}$	$1 = \frac{x^2+1}{2}$, $(\pm 1, 1)$	$3x^2 = \frac{x^2+1}{2}$, $5x^2 = 1$, $(\pm\frac{1}{\sqrt5}, \frac35)$

	$(0,1)$	$(0,0)$	$(1,1)$	$(-1,1)$	$(\frac{1}{\sqrt5}, \frac35)$	$(-\frac{1}{\sqrt5}, \frac{3}{\sqrt5})$
$f_{xx} = 6x - 6xy = 6x(1-y)$	0	0	0	0	$\frac{6}{\sqrt5}(1-\frac35)$	$\frac{-6}{\sqrt5}(1-\frac35)$
$f_{xy} = -3x^2 - 1 + 2y$	1	-1	-2	-2	$-\frac35 - 1 + \frac65 = \frac{-2}{5}$	$-\frac25$
$f_{yy} = 2x$	0	0	2	-2	$\frac{2}{\sqrt5}$	$\frac{-2}{\sqrt5}$
$D = f_{xx}f_{yy} - f_{xy}^2$	-1	-1	-4	-4	$\frac{12}{5}\cdot\frac25 - \frac{4}{25} > 0$	same
	saddles	saddles	saddles	saddles	f_{xx} pos MIN	f_{xx} neg MAX

2. $L = 3x + 4y - \lambda(x^2 + xy + y^2 - 1)$

$L_x = 3 - \lambda(2x + y)$ $\qquad L_x = 0 \qquad \lambda = \frac{3}{2x+y}$

$L_y = 4 - \lambda(x + 2y)$ $\qquad L_y = 0 \qquad \lambda = \frac{4}{x+2y}$

so at a C.P. $\frac{3}{2x+y} = \frac{4}{x+2y}$, i.e. $3x + 6y = 8x + 4y$

So $2y = 5x$

Intersect $y = \frac{5}{2}x$ and $x^2 + xy + y^2 = 1$:

$$x^2 + x\left(\tfrac{5}{2}x\right) + \tfrac{25}{4}x^2 = 1$$

$$\left(1 + \tfrac{5}{2} + \tfrac{25}{4}\right)x^2 = 1$$

$$\tfrac{39}{4}x^2 = 1$$

$$x^2 = \tfrac{4}{39} \qquad \text{so} \quad x = \pm\frac{2}{\sqrt{39}}$$

$$y = \tfrac{5}{2}x$$

C.P. $\left(\frac{2}{\sqrt{39}}, \frac{5}{\sqrt{39}}\right)$ and $\left(\frac{-2}{\sqrt{39}}, \frac{-5}{\sqrt{39}}\right)$.

Value $\frac{6}{\sqrt{39}} + \frac{20}{\sqrt{39}}$

$\boxed{= \frac{26}{\sqrt{39}}}$

$= \frac{2 \cdot 13}{\sqrt{3}\sqrt{13}}$

$= \frac{2\sqrt{13}}{\sqrt{3}}$

OR $\boxed{= 2\sqrt{\frac{13}{3}}}$

Max

Negative: $\boxed{\frac{-26}{\sqrt{39}}}$

$\boxed{= -2\sqrt{\frac{13}{3}}}$

Min