Name:______________________________

Math 2030, Fall 2017, Quiz 7
20 October 2017
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No calculators needed or allowed.

1. Find the $\boxed{\text{maximum and minimum values of } 3x+4y}$ on the circle $\boxed{x^2+y^2=1.}$
2. Find and classify the critical points of $f(x,y) = x^3 + xy^2 - 4x - y^2$.

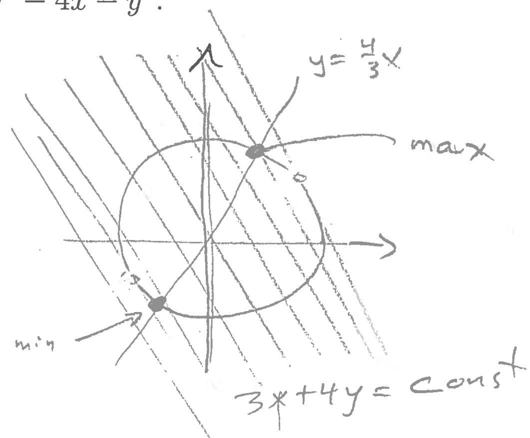
1

$$L = 3x + 4y - \lambda(x^2+y^2-1)$$

$$L_x = 3 - \lambda(2x)$$
$$L_y = 4 - \lambda(2y)$$
$$-L_\lambda = (x^2+y^2-1) = 0$$

C.P. $\Rightarrow$

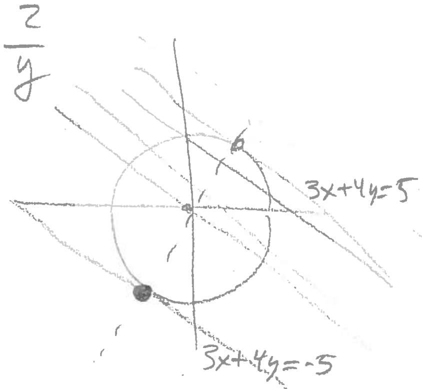
$3 = 2x\lambda$
$4 = 2y\lambda$

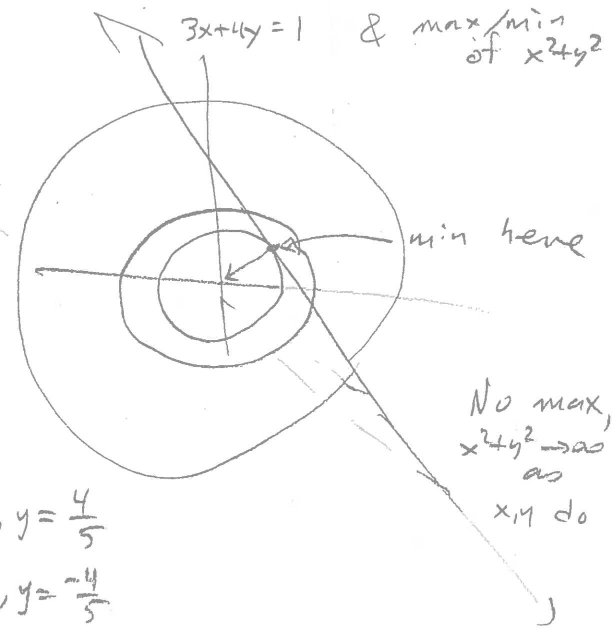


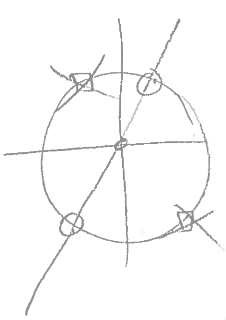
$$\lambda = \frac{3}{2x} = \frac{4}{2y} = \frac{2}{y}$$

$$\frac{2x}{3} = \frac{y}{2}$$

$$y = \frac{4x}{3}$$







$$x^2+y^2=1$$
$$x^2+\left(\frac{4}{3}x\right)^2=1$$
$$x^2+\frac{16}{9}x^2=1$$
$$\left(\frac{9+16}{9}\right)x^2=1$$
$$\frac{25}{9}x^2=1$$
$$x^2=\frac{9}{25}$$
$$x=\frac{3}{5} \text{ or } \frac{-3}{5}$$

$\Rightarrow y=\frac{4}{3}x$

at $x=\frac{3}{5}$, $y=\frac{4}{5}$

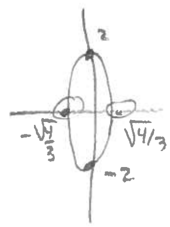
$x=\frac{-3}{5}$, $y=\frac{-4}{5}$

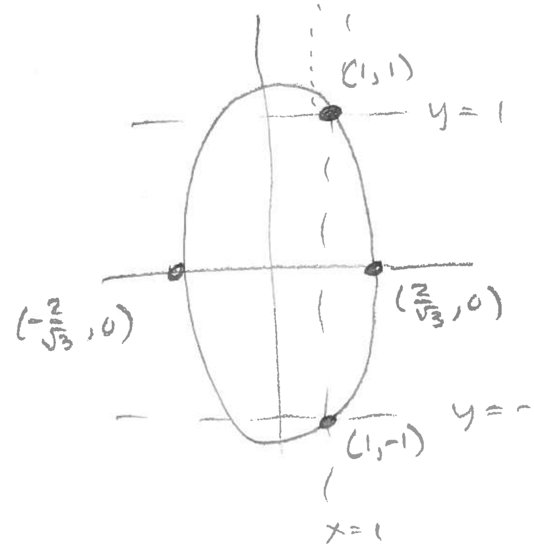
$\left(\frac{3}{5},\frac{4}{5}\right)$ $\quad 3x+4y = \frac{9}{5}+\frac{16}{5} = \frac{25}{5} = \boxed{5 \text{ MAX}}$

$\left(-\frac{3}{5},-\frac{4}{5}\right)$ $\quad 3x+4y = -\frac{25}{5} = \boxed{-5 \text{ MIN}}$

$f(x,y) = x^3 + xy^2 - 4x - y^2$

$f_x = 3x^2 + y^2 - 4$ UGH!





$f_y = 2xy - 2y$
$= 2y(x-1)$

$f_y = 0 \implies y = 0$ OR $x = 1$

If $y=0$ then $f_x = 3x^2 - 4 = 0$
when $x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

If $x=1$ then $f_x = 3 + y^2 - 4 = y^2 - 1 = 0$
when $y = \pm 1$

	$(\frac{2}{\sqrt{3}}, 0)$	$(\frac{-2}{\sqrt{3}}, 0)$	$(1,1)$	$(1,-1)$
$f_{xx} = 6x$	$\frac{12}{\sqrt{3}}$	$\frac{-12}{\sqrt{3}}$	6	6
$f_{xy} = 2y$	0	0	2	-2
$f_{yy} = 2x-2$	$2(\frac{2}{\sqrt{3}} - 1) > 0$	$2(\frac{-2}{\sqrt{3}} - 1)$	0	0
	pos * pos − 0 > 0	neg * neg − 0 > 0	$0 - 2^2$ < 0	$0 - (2y)^2$ < 0
	 min	 max	Saddle	Saddle