

Name: \_\_\_\_\_

Math 2030, Fall 2017, Quiz 4  
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No calculators needed or allowed.

For each of the following functions, decide whether

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y)$$

exists or not. If it exists, determine the limit.

1.  $f(x,y) = \frac{xy}{x^2+y^2}$

2.  $f(x,y) = \frac{xy^2}{x^2+y^2}$

3.  $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$

1. Along the line  $y=mx$  we have  $\frac{xy}{x^2+y^2} = \frac{mx^2}{(1+m^2)x^2} = \frac{m}{1+m^2}$   
which depends upon  $m$ , so the limit DNE.

2. Using  $x=r \cos \theta, y=r \sin \theta$ , we have  $\frac{xy^2}{x^2+y^2} = r \cos \theta \sin^2 \theta$ .  
This lies between  $\pm r$ ; since  $\pm r \rightarrow 0$  as  $(x,y) \rightarrow (0,0)$ ,

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^2} = 0.$$

3.  $\lim_{a \rightarrow 0} \frac{\sin(a)}{a} = 1$  and  $x^2+y^2 \rightarrow 0$  as  $(x,y) \rightarrow (0,0)$ , so

$$\lim_{(x,y) \rightarrow (0,0)} \left( \frac{\sin(x^2+y^2)}{x^2+y^2} \right) = 1.$$