Name: KEY

Math 2030, Fall 2017, Quiz 3
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No calculators needed or allowed.

Let $\vec{r}(t) = (2t, t + \dfrac{1}{t}, t^2)$.

1. Compute $\vec{v}(t)$.
2. Compute the tangent line $\vec{l}(t)$ at $t = 1$.
3. Compute $\vec{a}(t)$.
4. Show that $\text{proj}_{\vec{v}}(\vec{a}) = \text{proj}_{\vec{T}}(\vec{a})$.
   Hint: there is a nonzero constant c such that $\vec{v} = c\vec{T}$. Substitute this into the formula for $\text{proj}_{\vec{v}}(\vec{a})$.
5. Decompose $a(1)$ into tangential and normal components.
6. Compute ds/dt at $t = 1$.
7. Compute the curvature κ and radius of curvature R at $t = 1$.
8. Compute the osculating circle at $t = 1$, or at least say as much about it as you can.

1. $\boxed{\vec{v}(t) = (2, 1 - \frac{1}{t^2}, 2t)}$

2. $\vec{r}(1) = (2, 2, 1)$
   $\vec{v}(1) = (2, 0, 2)$
   $\boxed{\vec{l}(t) = (2,2,1) + t(2,0,2)}$

3. $\boxed{\vec{a}(t) = (0, \frac{2}{t^3}, 2)}$

4. $\text{proj}_{\vec{v}} \vec{a} = \dfrac{\vec{a}\cdot\vec{v}}{\vec{v}\cdot\vec{v}} \vec{v}$
   $= \dfrac{\vec{a}\cdot c\vec{T}}{c\vec{T}\cdot c\vec{T}} c\vec{T}$
   $= \dfrac{c^2}{c^2}\left(\dfrac{\vec{a}\cdot\vec{T}}{\vec{T}\cdot\vec{T}} \vec{T}\right) = \text{proj}_{\vec{T}} \vec{a}$

5. $\vec{a}(1) = (0, 2, 2)$ so
   $\boxed{\vec{a}_T = \text{proj}_{\vec{v}(1)} \vec{a}(1) = \dfrac{(0,2,2)\cdot(2,0,2)}{(2,0,2)\cdot(2,0,2)} (2,0,2) = \dfrac{4}{8}(2,0,2) = (1,0,1)}$
   $\boxed{\vec{a}_N} = \vec{a} - \vec{a}_T = (0,2,2) - (1,0,1) = \boxed{(-1,2,1)}$
   so that $\vec{a}(1) = \vec{a}_T + \vec{a}_N$

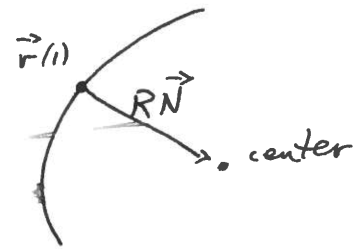
6. $\left.\dfrac{ds}{dt}\right|_{t=1} = |\vec{v}(1)| = \sqrt{2^2+0^2+2^2} = \boxed{\sqrt{8}}$

7. $\kappa = |\vec{a}_N| / v^2 = \dfrac{\sqrt{1+4+1}}{8} = \dfrac{\sqrt{6}}{8}$

8. see reverse.

8. $R = \frac{1}{\kappa} = \frac{8}{\sqrt{6}}$

$\vec{N} = \frac{\vec{a}_N}{|\vec{a}_N|} = \frac{(-1,2,1)}{\sqrt{6}}$



So the center of the osculating circle is $\left[\frac{8}{\sqrt{6}} \frac{(-1,2,1)}{\sqrt{6}}\right]$

$\vec{r}(1) + R\vec{N} = (2,2,1) + \frac{8}{\sqrt{6}} \frac{(-1,2,1)}{\sqrt{6}}$ FIX BLUR

$= (2,2,1) + \frac{4}{3}(-1,2,1) = \left(2-\frac{4}{3}, 2+\frac{8}{3}, 1+\frac{4}{3}\right)$

$= \left(\frac{2}{3}, \frac{14}{3}, \frac{7}{3}\right)$

Now the challenge: how do you write a circle in space?

We know one radius ought to be $-R\vec{N}$ to get from the center to $\vec{r}(1)$ on the curve. Also, we want to use $R\vec{T}$ to move tangent to the curve.

Key: If P is a point in $\mathbb{R}^3$ (or $\mathbb{R}^n$)

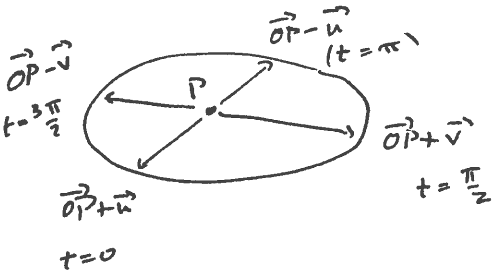
and $\vec{u}$ and $\vec{v}$ are vectors such that

$|\vec{u}| = |\vec{v}|$ (= R, let's say)

and $\vec{u} \perp \vec{v}$

then $\boxed{\vec{r}(t) = \vec{OP} + (\cos t)\vec{u} + (\sin t)\vec{v}}$

is a circle in the plane spanned by $\vec{u}$ and $\vec{v}$ (passing through, or from, P) with center P.



Here $R\vec{T} = \frac{8}{\sqrt{6}} \frac{\vec{v}}{v} = \frac{8}{\sqrt{6}} \frac{1}{\sqrt{8}}(2,0,2) = \sqrt{\frac{4}{3}}\,(2,0,2)$

So $\vec{r}(t) = \left(\frac{2}{3}, \frac{14}{3}, \frac{7}{3}\right) + (\cos t)\left(\frac{4}{3}(-1,2,1)\right) + (\sin t)\left(\sqrt{\frac{4}{3}}\,(2,0,2)\right)$