

Name: KEY

Math 2030, Fall 2017, Quiz 2

Scale: 3 pts each Q.

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No calculators needed or allowed.

Let $P_0 = (1, 3, 4)$, $P_1 = (2, 2, 5)$, and $P_2 = (3, 2, 7)$.

Let $Q = (-1, 0, 8)$.

1. Compute the vectors $\vec{u} = P_0P_1$ and $\vec{v} = P_1P_2$.
2. Compute the cross product $\vec{n} = \vec{u} \times \vec{v}$.
3. Find an equation for the plane through P_0 , P_1 and P_2 .
4. Find an equation for the line through Q in the direction of \vec{n} .
5. Find the intersection of that line with the plane from the preceding problem.
6. How far is Q from the plane?

$$1. \quad \vec{u} = \overrightarrow{P_0P_1} = P_1 - P_0 = (1, -1, 1)$$

$$\vec{v} = \overrightarrow{P_1P_2} = P_2 - P_1 = (1, 0, 2)$$

$$2. \quad \vec{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = \overrightarrow{(-2, -(2-1), 1)} \\ = \overrightarrow{(-2, -1, 1)}$$

$$3. \quad -2x - y + z = -2 - 3 + 4 = -1$$

$$-2x - y + z = -1$$

$$4. \quad \vec{r}(t) = Q + t\vec{n}$$

$$= (-1, 0, 8) + (-2t, -t, t)$$

$$= (-1-2t, -t, 8+t)$$

$$5. \quad -2(-1-2t) - (-t) + (8+t) = -1$$

$$2 + 4t + t + 8 + t = -1$$

$$6t = -11$$

$$t = -\frac{11}{6}$$

$$\text{Int. point} = \left(-1 + \frac{22}{6}, \frac{11}{6}, 8 - \frac{11}{6}\right)$$

$$5. \text{ (cont.) Int. point} = \left(\frac{16}{6}, \frac{11}{6}, \frac{37}{6}\right)$$

6. The shortest vector from Q to the plane is $-\frac{11}{6}(-2, -1, 1) = \frac{11}{6}(2, 1, -1)$. Its length, $\frac{11}{6}\sqrt{4+1+1} = \frac{11}{\sqrt{6}}$, is the distance from Q to the plane.

