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Math 2030, Winter 2017, Quiz 13
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No calculators needed or allowed.

Let $\mathcal{S}$ be the surface $\mathbf{r}(x, y) = (x, y, x^2 + y^2)$ over the disk $x^2 + y^2 \leq 1$.
Let $\mathbf{F}(x, y, z) = (y, 0, z)$.

1. Compute $\mathbf{n}\, dS = \dfrac{\partial \mathbf{r}}{\partial x} \times \dfrac{\partial \mathbf{r}}{\partial y}\, dA$.
2. Compute $\nabla \times \mathbf{F}$.
3. Compute $\displaystyle\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$.

Let $\mathbf{r}(\theta) = (\cos(\theta), \sin(\theta), 1)$, $0 \leq \theta \leq 2\pi$, be the boundary, ∂S.

4. Compute $\vec{T} ds = d\mathbf{r} = \dfrac{d\mathbf{r}}{d\theta}\, d\theta$.
5. Compute $\displaystyle\int_{\partial S} \mathbf{F} \cdot \vec{T} ds$.

(1.) $\vec{n}\, dS = \vec{r}_x \times \vec{r}_y\, dx\, dy = \begin{vmatrix} i & j & k \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} dx\, dy = \boxed{(-2x, -2y, 1)\, dx\, dy}$

(2.) $\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & 0 & z \end{vmatrix} = \boxed{(0, 0, -1)}$

(3.) $\displaystyle\iint_{\substack{\text{unit}\\\text{disk}}} (0,0,-1)\cdot(-2x,-2y,1)\, dx\, dy = \iint_{\substack{\text{unit}\\\text{disk}}} -1\, dA = -\text{Area(Disk)} = \boxed{-\pi}$

(4.) $\vec{T} ds = (dx, dy, dz) = \boxed{(-\sin\theta, \cos\theta, 0)\, d\theta}$

(5.) $\displaystyle\int_{\partial S} \vec{F} \cdot \vec{T} ds = \int_0^{2\pi} (+\sin\theta, 0, 1)\cdot(-\sin\theta, \cos\theta, 0)\, d\theta = \int_0^{2\pi} -\sin^2\theta\, d\theta$

$\displaystyle = -\frac{\theta - \sin\theta\cos\theta}{2}\Bigg|_0^{2\pi} = -\left(\frac{2\pi - 0}{2}\right) + \left(\frac{0-0}{2}\right) = \boxed{-\pi}$

They agree, as they should.