Name:______________________________

Math 2030, Fall 2017, Quiz 11
10 November 2017
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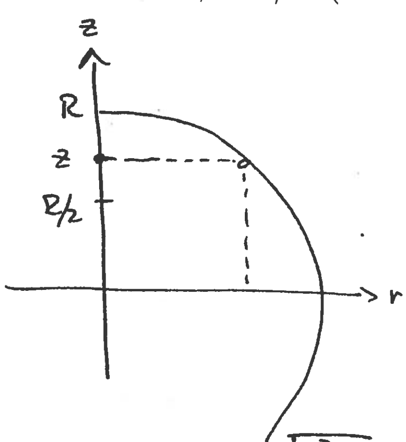
No calculators needed or allowed.

Let D be the region inside the sphere of radius R centered at the orgin and above the plane $z = R/2$.

(6) 1. Describe D in cylindrical coordinates.

(10) 2. Compute the volume of D.

(6) 3. Compute the volume of that part of the sphere which lies between angles $\phi = \pi/6$ and $\phi = \pi/3$. (This is not related to D.)



1.

$$\left.\begin{aligned} 0 &\le \theta \le 2\pi \\ R/2 &\le z \le R \\ 0 &\le r \le \sqrt{R^2-z^2} \end{aligned}\right\}$$

1 point for each limit

from $\int_0^{2\pi} d\theta$

2. $$\int_0^{2\pi}\int_{R/2}^{R}\int_0^{\sqrt{R^2-z^2}} r\,dr\,dz\,d\theta = 2\pi \int_{R/2}^{R} \left.\tfrac{1}{2}r^2\right|_0^{\sqrt{R^2-z^2}} dz = \pi\int_{R/2}^{R} R^2-z^2\,dz$$

$$\doteq \pi\left[R^2z - \left.\frac{z^3}{3}\right|_{R/2}^{R}\right] = \pi\left(R^3 - \frac{R^3}{3} - \left(\frac{R^3}{2} - \frac{R^3}{24}\right)\right) = \pi R^3\left(1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{24}\right) = \frac{5\pi}{24}R^3$$

4 points: correct initial integral
+ 5 points: get to here
+ last point:

10 total

3. OVER

3. $$\int_0^{2\pi}\int_{\pi/6}^{\pi/3}\int_0^{R} \rho^2 \sin\phi \; d\rho \, d\phi \, d\theta$$ ← 3 points if they get this far

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_{\pi/6}^{\pi/3} \sin\phi \, d\phi\right)\left(\int_0^{R} \rho^2 \, d\rho\right)$$

interpolate sensibly :)

$$= 2\pi \left(-\cos\left(\frac{\pi}{3}\right) + \cos\frac{\pi}{6}\right)\left(\frac{R^3}{3}\right)$$

$$= \frac{2\pi R^3}{3}\left(\frac{\sqrt{3}-1}{2}\right) = \pi R^3\left(\frac{\sqrt{3}-1}{3}\right)$$ ← 6 points