Name:______________________________

Math 2030, Fall 2017, Quiz 10
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R. Bruner

No calculators needed or allowed.

Let $(x, y) = F(u, v)$ be the transformation $x = u - v^2$, $y = v - u^2$, from $\mathbf{R}^2$ to $\mathbf{R}^2$.
Let $D = [1, 2] \times [1, 2]$ be the indicated rectangle in the uv-plane.

The image $F(D)$ in the xy-plane is bounded by parabolas, with 'vertices' the points $F(1,1) = (0,0)$, $F(1,2) = (-3,1)$, $F(2,2) = (-2,-2)$, and $F(2,1) = (1,-3)$.

(5) 1. Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

(10) 2. Compute the area

$$\text{Area}(F(D)) = \iint_{F(D)} dx\,dy.$$

(5) 3. Compute the x-coordinate of the centroid

$$\overline{x} = \frac{1}{\text{Area}(F(D))} \iint_{F(D)} x\;dx\,dy$$

20 total possible

1. $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & -2v \\ -2u & 1 \end{vmatrix} = \boxed{1-4uv}$ 5 pts

2. $\displaystyle\iint_{F(D)} dx\,dy = \iint_D 1-4uv\;du\,dv = \boxed{\int_1^2\int_1^2 1-4uv\;du\,dv}$ 5 pts

$\displaystyle = \int_1^2 u - 2u^2v\Big|_{u=1}^{u=2} dv = \int_1^2 2-8v-(1-2v)\,dv = \int_1^2 1-6v\,dv = v-3v^2\Big|_{v=1}^{v=2}$

$= 2-12-(1-3) = -10+2 = \boxed{-8}$ (4 pts)

The negative sign reflects orientation reversal.

$\text{Area}(F(D)) = \boxed{8}$.

Getting the sign right: $\boxed{1}$ more pt

Tot for (2): 10 pts

20

3. $$\iint_{F(D)} x\,dx\,dy = \int_1^2\int_1^2 (u-v^2)(1-4uv)\,du\,dv$$ $\boxed{5\text{ pts}}$

I did not ask them to go any further.

$$= \int_1^2\int_1^2 u - v^2 - 4u^2v + 4uv^3\,du\,dv$$

$$= \int_1^2 \frac{1}{2}u^2 - uv^2 - \frac{4}{3}u^3v + 2u^2v^3 \Big|_{u=1}^{u=2} dv$$

$$= \int_1^2 \begin{array}{l} 2 - 2v^2 - \frac{32}{3}v + 8v^3 \\ -\frac{1}{2} + v^2 + \frac{4}{3}v - 2v^3 \end{array} dv$$

$$= \int_1^2 \frac{3}{2} - v^2 - \frac{28}{3}v + 6v^3\,dv$$

$$= \frac{3}{2}v - \frac{1}{3}v^3 - \frac{14}{3}v^2 + \frac{6}{4}v^4 \Big|_{v=1}^{v=2}$$

$$= \frac{3}{2}(2) - \frac{1}{3}(8) - \frac{14}{3}(4) + \frac{6}{4}(16) - \left(\frac{3}{2} - \frac{1}{3} - \frac{14}{3} + \frac{6}{4}\right)$$

$$= \begin{array}{l} 3 - \frac{8}{3} - \frac{56}{3} + 24 \\ -\frac{3}{2} + \frac{1}{3} + \frac{14}{3} - \frac{3}{2} \end{array}$$

$$= 24 - \frac{49}{3} = \frac{72-49}{3} = \frac{23}{3}$$

Hmmm... Last time I got $\frac{-22}{3}$.

$$\bar{x} = \frac{23/3}{-8} = \frac{-23}{24} \quad \left[= \bar{y}, \text{ by symmetry} \right]$$