Name:________________________________

Math 2030, Winter 2017, Quiz 14
11 December 2017
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No calculators needed or allowed.

Let $\mathcal{V}$ be the cylindrical solid given by $0 \leq z \leq 4$ and $0 \leq r \leq 2$ in cylindrical coordinates. Observe that the boundary surface of $\mathcal{V}$ has three parts: a top disk $\mathcal{T}_1$, a bottom disk $\mathcal{T}_0$, and the lateral side surface $\mathcal{S}$ given by $r = 2$, $0 \leq z \leq 4$.

Let $\mathbf{F}(x, y, z) = (x, y, 0)$.

1. Show that $\mathbf{F} \cdot \mathbf{n} = 0$ on $\mathcal{T}_0$ and $\mathcal{T}_1$, so that
$$\iint_{\partial\mathcal{V}} \mathbf{F} \cdot \mathbf{n}\, \mathbf{dS} = \iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n}\, \mathbf{dS}$$
2. On a surface of revolution $r = f(z)$, we have $\mathbf{n}\; dS = (x, y, -f(z)f'(z))\, dz d\theta$. Use this to compute $\mathbf{n}\; dS$ for the lateral surface $\mathcal{S}$.
3. Compute $\displaystyle\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n}\; dS$.
4. Compute $\nabla \cdot \mathbf{F}$.
5. Compute $\displaystyle\iiint_{\mathcal{V}} \nabla \cdot \mathbf{F}\; dV$.