Math 2030, Fall 2017, Quiz 10
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No calculators needed or allowed.

Let $(x, y) = F(u, v)$ be the transformation $x = u - v^2$, $y = v - u^2$, from $\mathbf{R}^2$ to $\mathbf{R}^2$.
Let $D = [1, 2] \times [1, 2]$ be the indicated rectangle in the uv-plane.

The image $F(D)$ in the xy-plane is bounded by parabolas, with 'vertices' the points $F(1, 1) = (0, 0)$, $F(1, 2) = (-3, 1)$, $F(2, 2) = (-2, -2)$, and $F(2, 1) = (1, -3)$.

1. Compute the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

2. Compute the area
$$\text{Area}(F(D)) = \iint_{F(D)} dx\, dy.$$

3. Compute the x-coordinate of the centroid
$$\overline{x} = \frac{1}{\text{Area}(F(D))} \iint_{F(D)} x\, dx\, dy$$