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Math 2030, Fall 2017, Final Exam
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Each problem is worth 10 points unless otherwise indicated.

1. (10) Find and classify the critical points of $f(x, y) = 4x^3 - 2x^2y + y^2 + 8y$.

2. Consider the ellipse $4x^2 + y^2 = 4$.

   (a) (10) Find the absolute maximum and minimum value of $x + 2y$ on the ellipse.

   (b) (5) Find the tangent line to the ellipse at the point where the maximum value of $x + 2y$ occurs.

3. (15) Let C be the closed curve $(x, y) = (1 - t^2, t^3 - t)$, $-1 \leq t \leq 1$.

   (a) Compute $\int_C x\,dy$.

   (b) Compute $\int_C y\,dx$.

   (c) What is the area inside C? Why?

4. (24) Let $\mathbf{r}(t) = (t^2 + 4t, t^3 - 8t)$.

   (a) Find the velocity $\mathbf{v} = \mathbf{r}'$.

   (b) Find the speed at time $t = 2$.

   (c) Find the tangent line to $\mathbf{r}(t)$ at $t = 2$.

   (d) Find the acceleration $\mathbf{a} = \mathbf{r}''$.

   (e) Decompose $\mathbf{a}(2)$ into tangential and normal components.

   (f) Compute the curvature κ at that point.

5. (10) Find all first and second partial derivatives of $e^x \cos(2y)$.

6. (10) Find the tangent plane to the surface $z = x^2y + xy^3$ at the point $(x, y) = (1, 2)$.

7. (15) Let $f(x, y, z) = xyz + x^2 + y^2 + z^2$.

   (a) In which direction does f increase most rapidly at the point $(x, y, z) = (3, 2, 1)$?

   (b) What is the tangent plane to the level surface of f at $(3, 2, 1)$?

   (c) Compute $\partial z/\partial x$ and $\partial z/\partial y$ if $z(x, y)$ is determined by the level surfaces of f.

8. (10) Decompose $(4, 3, 4)$ as the sum of a vector parallel to $(1, 2, 2)$ and a vector perpendicular to $(1, 2, 2)$.

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9. (10) Determine whether the following vector fields are conservative or not. Find a potential function for each conservative field.

   (a) $\vec{F}(x, y, z) = (y, x, z)$

   (b) $\vec{F}(x, y, z) = (y, z, x)$

10. (10) Compute $\int_C \nabla f \cdot d\vec{r}$, i.e. $\int_C \nabla f \cdot \vec{T}\, ds$, where $f(x, y) = x^2 y$ and C is a curve which starts at $(2, 3)$ and ends at $(1, 2)$.

11. (10) Let B be the region given by $0 \leq r \leq z - z^3$ and $0 \leq z \leq 1$ in cylindrical coordinates. Find the volume of B.

Recall that if a surface $\mathcal{S}$ is given by $r = f(z)$ in cylindrical coordinates, then

$$d\vec{S} = \vec{n} dS = (x, y, -f(z)f'(z))\ dz\, d\theta,$$

where x and y are expressed as usual in cylindrical coordinates using $r = f(z)$.

12. (15) Let $\mathcal{S}$ be the surface $r = 1 - z$ for $0 \leq z \leq 1$. Consider the vector field $\vec{F}(x, y, z) = (x, y, 0)$.

   (a) Compute $\iint_{\mathcal{S}} \vec{F} \cdot \vec{n}\ dS$.

   (b) Let $\mathcal{V}$ be the solid cone bounded by $\mathcal{S}$ and the xy-plane. Show that $\vec{F} \cdot \vec{n} = 0$ when $\vec{n}$ is a normal vector to that part of $\partial\mathcal{V}$ which lies in the xy-plane.

   (c) Compute $\iiint_{\mathcal{V}} \nabla \cdot \vec{F}\ dV$.

13. (10) Let R be a disk of radius 2. If $Q_x - P_y = 3$, evaluate $\int_{\partial R} P\, dx + Q\, dy$.

14. (15) Let $\mathcal{S}$ be the part of the surface $z = 2x + 4y + 1$ which lies inside the cylinder $x^2 + y^2 = 1$. Let $\mathbf{F}(x, y, z) = (y, y, 0)$.

   (a) Find the surface area of $\mathcal{S}$.

   (b) Compute $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \cdot d\mathbf{S}$.

   (c) Compute $\int_{\partial\mathcal{S}} \mathbf{F} \cdot d\mathbf{r}$. (Hint: You may use Stokes' Theorem.)

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