Solutions to Test 2 Math 2020

1. $\int (2x-5)e^{3x}\,dx = \frac{1}{3}(2x-5)e^{3x} - \int \frac{2}{3}e^{3x}\,dx$

$u = 2x-5 \qquad du = 2\,dx$

$dv = e^{3x}dx \qquad v = \frac{1}{3}e^{3x}$

$$\boxed{= \frac{1}{3}(2x-5)e^{3x} - \frac{2}{9}e^{3x} + C}$$

2. $\int \sin^4(\theta)\cos^3\theta\,d\theta = \int u^4(1-u^2)\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$\boxed{= \frac{1}{5}\sin^5\theta - \frac{1}{7}\sin^7\theta + C}$$

3. $\int \frac{x+1}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$

$u = x^2+4 \qquad u = \frac{x}{2}$

$\frac{1}{2}du = x\,dx \qquad 2\,du = dx$

$$= \frac{1}{2}\int \frac{du}{u} + \frac{2}{4}\int \frac{du}{u^2+1} = \boxed{\frac{1}{2}\ln|x^2+4| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

4. $\int \frac{x^2}{x^2-2x+2}\,dx = \int \frac{x^2-2x+2+2x-2}{x^2-2x+2}\,dx = \int 1 + \frac{2x-2}{x^2-2x+2}\,dx$

$$\boxed{= x + \ln|x^2-2x+2| + C}$$

$u = x^2-2x+2$

$du = (2x-2)\,dx$

5. $$\frac{x}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

$$x = A(x+4) + B(x-2)$$

$x=2$ $\quad 2 = 6A \quad A = 1/3$

$x=-4$ $\quad -4 = -6B \quad B = 2/3$

$$\int \frac{x}{(x-2)(x+4)}\,dx = \boxed{\tfrac{1}{3}\ln|x-2| + \tfrac{2}{3}\ln|x+4| + C}$$

6. $$\frac{x}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$$

$$x = A(x^2-2x+4) + (Bx+C)(x+2)$$

$x=-2$ $\quad -2 = 12A \quad$ so $\quad A = -1/6$

$x=0$ $\quad 0 = 4A + 2C \quad$ so $\quad C = -2A = 1/3$

$x=1$ $\quad 1 = 3A + (B+C)(3)$

$\quad 1 = -\frac{1}{2} + 3B + 1$

$\quad \frac{1}{2} = 3B \qquad B = 1/6$

$$\boxed{\frac{x}{(x+2)(x^2-2x+4)} = \frac{-1/6}{x+2} + \frac{\frac{1}{6}x + \frac{1}{3}}{x^2-2x+4}}$$

7. $$\int x\sqrt{9-4x^2}\,dx = -\frac{1}{8}\int \sqrt{u}\,du = -\frac{1}{8}\left(\frac{2}{3}\right)u^{3/2}+c$$

$u = 9-4x^2$

$du = -8x\,dx$

$-\frac{1}{8}du = x\,dx$

$$= \boxed{\frac{-1}{12}(9-4x^2)^{3/2}+C}$$

8. $$\int \frac{dx}{\sqrt{16-25x^2}} = \int \frac{dx}{4\sqrt{1-\left(\frac{5x}{4}\right)^2}} = \frac{4}{5}\,\frac{1}{4}\int \frac{\cos\theta\,d\theta}{\cos\theta}$$

$\frac{5x}{4} = \sin\theta$

$\frac{5}{4}dx = \cos\theta\,d\theta$

$\sqrt{1-\left(\frac{5x}{4}\right)^2} = \sqrt{1-\sin^2\theta} = \cos\theta$

$$= \frac{1}{5}\theta + C$$

$$\boxed{= \frac{1}{5}\sin^{-1}\left(\frac{5x}{4}\right)+C}$$

9. $$\int_1^\infty \frac{\ln x}{x^2}\,dx = uv - \int v\,du\Big|_1^\infty = \frac{-\ln x}{x} + \int x^{-2}\,dx\Big|_1^\infty$$

$u = \ln x \qquad du = x^{-1}dx$

$dv = x^{-2}dx \qquad v = -x^{-1}$

$$= \frac{-\ln x}{x} - \frac{1}{x}\Big|_1^\infty$$

$$= \lim_{A\to\infty}\left(\frac{-\ln A}{A} - \frac{1}{A} + \left(\frac{0}{1}+1\right)\right) = \boxed{1}$$

7. (Alternate version) (i.e. different method)

$$\int x\sqrt{9-4x^2}\,dx = \int \left(\tfrac{3}{2}\sin\theta\right)\left(3\cos\theta\right)\left(\tfrac{3}{2}\cos\theta\right)d\theta$$

$x = \frac{3}{2}\sin\theta$

$dx = \frac{3}{2}\cos\theta\, d\theta$

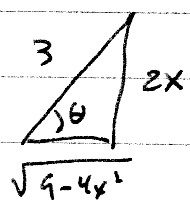
$$= \frac{27}{4}\int \sin\theta\cos^2\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= -\frac{27}{4}\int u^2\,du$$

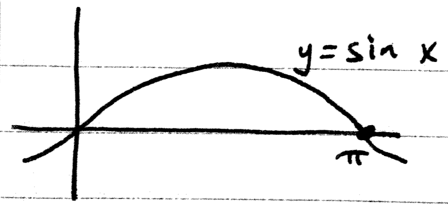
$$= -\frac{9}{4}u^3 + C$$



$$= -\frac{9}{4}\cos^3\theta + C$$

$$= -\frac{9}{4}\left(\frac{\sqrt{9-4x^2}}{3}\right)^3 + C$$

$$= -\frac{1}{12}\left(9-4x^2\right)^{3/2} + C$$

10. 

$$\text{Area} = \int_0^{\pi} \sin x\, dx$$

$$= -\cos x \Big|_0^{\pi} = -(-1) + 1 = \boxed{2}$$

$\bar{x}$ needs $\displaystyle\int_0^{\pi} x \sin x\, dx = -x\cos x + \int \cos x\, dx \Big|_0^{\pi}$

$u = x \qquad du = dx$

$dv = \sin x\, dx \qquad v = -\cos x$

$$= -x\cos x + \sin x \Big|_0^{\pi} = -\pi(-1) + 0 - (-0 + 0)$$

$$= \pi$$

so $\boxed{\bar{x} = \pi/2}$

$\bar{y}$ needs $\displaystyle\frac{1}{2}\int_0^{\pi} \sin^2 x\, dx = \frac{1}{4}\int_0^{\pi} 1 - \cos 2\theta\, d\theta$ (oops! $\theta = x$)

$$= \frac{1}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi} = \frac{1}{4}\left(\pi - 0 - (0 - 0)\right) = \pi/4$$

so $\boxed{\bar{y} = \pi/8}$

$\boxed{\text{Centroid}: \left(\frac{\pi}{2}, \frac{\pi}{8}\right)}$ $\boxed{\text{Area} = 2}$

2 ea Area, M_x, M_y

2 ea formula for $\bar{x}, \bar{y}$