

R. Bruner
Math 2020, Winter 2007, Quiz 8
28 February 2007

Consider the curve $x(t) = t^4 - t$, $y(t) = t^4 - t^3$.

1. Compute dy/dx .
2. Where is the curve horizontal?
3. Compute the area inside the loop formed by the segment between $t = 0$ and $t = 1$.

$$1. \frac{dx}{dt} = 4t^3 - 1, \quad \frac{dy}{dt} = 4t^3 - 3t^2 \quad \text{so} \quad \boxed{\frac{dy}{dx} = \frac{4t^3 - 3t^2}{4t^3 - 1}}$$

$$2. \text{Horizontal} \Leftrightarrow \frac{dy}{dt} = 0 \Leftrightarrow 4t^3 - 3t^2 = 0$$
$$\Leftrightarrow t^2(4t - 3) = 0$$

→
←

$$\Leftrightarrow \boxed{t = 0 \text{ OR } t = 3/4}$$

$$3. \int_{t=0}^{t=1} y dx = \int_0^1 (t^4 - t^3)(4t^3 - 1) dt$$
$$= \int_0^1 4t^7 - t^4 - 4t^6 + t^3 dt = \frac{4}{8} - \frac{1}{5} - \frac{4}{7} + \frac{1}{4}$$
$$= \frac{1}{2} - \frac{1}{5} - \frac{4}{7} + \frac{1}{4} = \frac{70 - 28 - 80 + 35}{140}$$
$$= \frac{105 - 108}{140} = \frac{-3}{140} \quad \boxed{\text{Area} = \frac{3}{140}}$$