R. Bruner
Math 2020, Winter 2007, Quiz 7
21 February 2007

Let R be the region
$$\left\{\begin{array}{ccccc} 1 & \leq & x & < & \infty \\ 0 & \leq & y & \leq & 1/x^3 \end{array}\right\}$$
Compute the area of R and the centroid $(\bar{x}, \bar{y})$ of R.

$$\text{Area} = \int_{\phi}^{\infty} \frac{1}{x^3}\,dx = \lim_{A\to\infty}\left(\frac{-1}{2x^2}\Big|_1^A\right) = \lim_{A\to\infty}\left(\frac{-1}{2A^2}+\frac{1}{2}\right)$$

$$= \frac{1}{2}$$

$$\text{For } \bar{x}: \quad \int_1^{\infty}\frac{x}{x^3}\,dx = \lim_{A\to\infty} \frac{-1}{x}\Big|_1^A = \lim_{A\to\infty}\left(\frac{-1}{A}+1\right)$$

$$= 1 \quad \text{so} \quad \bar{x} = \frac{1}{1/2} = 2$$

$$\text{For } \bar{y}: \quad \frac{1}{2}\int_1^{\infty}\left(\frac{1}{x^3}\right)^2 dx = \frac{1}{2}\lim_{A\to\infty}\frac{-1}{5x^5}\Big|_1^A = \frac{1}{2}\lim_{A\to\infty}\left(\frac{-1}{5A^5}+\frac{1}{5}\right)$$

$$= \frac{1}{10} \quad \text{so} \quad \bar{y} = \frac{1/10}{1/2} = \frac{1}{5}$$

$$\text{Centroid} = \left(2, \frac{1}{5}\right) \qquad \text{Area} = \frac{1}{2}$$