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Math 2020, Winter 2007, Quiz 6
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Compute $\int \frac{x}{(x-1)(x^2+4)}\,dx$

$$\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$x = A(x^2+4) + (Bx+C)(x-1)$$

$x=0$ $\quad 0 = 4A - C$ so $C = 4A$

$x=1$ $\quad 1 = 5A$ so $A = 1/5$, $C = 4/5$

$x=2$ $\quad 2 = 8A + (2B+C)$ so

$2B = 2 - 8/5 - 4/5 = -2/5$

$B = -1/5$

$$\frac{1}{5}\int \frac{1}{x-1}\,dx = \frac{1}{5}\ln|x-1| + C$$

$$-\frac{1}{5}\int \frac{x-4}{x^2+4}\,dx = -\frac{1}{5}\int \frac{x}{x^2+4} + \frac{4}{5}\int \frac{1}{x^2+4}\,dx$$

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|x^2+4| + C$$

$u = x^2+4$

$\frac{1}{2}du = x\,dx$

$$\int \frac{1}{x^2+4}\,dx = \frac{1}{4}\int \frac{dx}{(\frac{x}{2})^2+1} = \frac{2}{4}\int \frac{du}{u^2+1} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$u = \frac{x}{2}$

$2\,du = dx$

continuing from the left column:

$$\int \frac{x}{(x-1)(x^2+4)}\,dx =$$

$$\frac{1}{5}\int \frac{1}{x-1}\,dx$$

$$-\frac{1}{5}\int \frac{x}{x^2+4}\,dx$$

$$+\frac{4}{5}\int \frac{1}{x^2+4}\,dx$$

$$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln|x^2+4|$$

$$+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + C$$