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 Math 2020, Winter 2007, Quiz 5
 7 February 2007

Do both of the following:

$$1. \int \sin^3(x) \cos^2(x) dx$$

$$2. \int \sqrt{4 - 9x^2} dx$$

$$1. -du = \sin x \, dx$$

$$u = \cos x$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$$

$$2. \int \sqrt{4 - 9x^2} dx = \int 2\cos \theta \cdot \frac{2}{3}\cos \theta d\theta$$

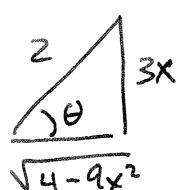
$$\text{Want: } 4 - 9x^2 = 4 - 4\sin^2 \theta$$

$$\text{so let } x = \frac{2}{3}\sin \theta$$

$$dx = \frac{2}{3}\cos \theta d\theta$$

$$\sqrt{4 - 9x^2} = 2\cos \theta$$

$$\sin \theta = \frac{3x}{2}$$



$$\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x (\sin x \, dx)$$

$$= \int (1 - u^2) u^2 (-du) = \int u^4 - u^2 du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$$

$$= \frac{4}{3} \int \cos^2 \theta d\theta$$

$$= \frac{4}{3} \left(\frac{\theta}{2} + \frac{\sin \theta \cos \theta}{2} \right) + C$$

$$= \frac{2}{3} \left(\sin^{-1} \left(\frac{3x}{2} \right) + \frac{3x}{2} \frac{\sqrt{4 - 9x^2}}{2} \right) + C$$

$$= \boxed{\frac{2}{3} \sin^{-1} \left(\frac{3x}{2} \right) + \frac{1}{2} x \sqrt{4 - 9x^2} + C}$$