R. Bruner

Math 2020, Winter 2007, Quiz 5

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Do both of the following:

1. $\int \sin^3(x)\cos^2(x)\,dx$

2. $\int \sqrt{4-9x^2}\,dx$

1. $-du = \sin x\,dx$

$u = \cos x$

$$\int \sin^3 x \cos^2 x\,dx = \int \sin^2 x \cos^2 x\,(\sin x\,dx)$$

$$= \int (1-u^2)u^2(-du) = \int u^4 - u^2\,du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$$

2. $$\int \sqrt{4-9x^2}\,dx = \int 2\cos\theta \cdot \frac{2}{3}\cos\theta\,d\theta$$

Want: $4-9x^2 = 4-4\sin^2\theta$

so let $x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\,d\theta$

$\sqrt{4-9x^2} = 2\cos\theta$

$$= \frac{4}{3}\int \cos^2\theta\,d\theta$$

$$= \frac{4}{3}\left(\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}\right) + C$$

$$= \frac{2}{3}\left(\sin^{-1}\left(\frac{3x}{2}\right) + \frac{3x}{2}\,\frac{\sqrt{4-9x^2}}{2}\right) + C$$

$$\boxed{= \frac{2}{3}\sin^{-1}\left(\frac{3x}{2}\right) + \frac{1}{2}x\sqrt{4-9x^2} + C}$$

$\sin\theta = \frac{3x}{2}$