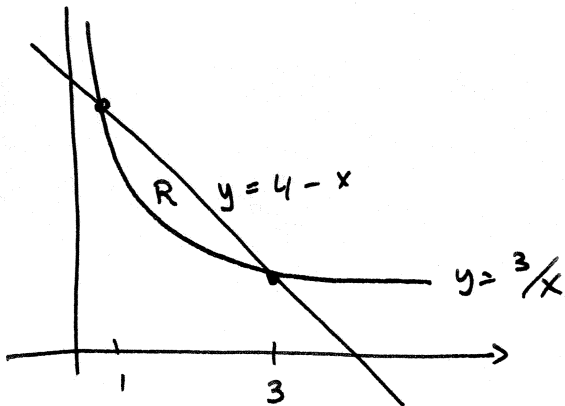


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Let R be the region in the first quadrant which is below the line $y = 4 - x$ and above the curve $y = 3/x$.

1. Find the area of R .
2. Find the volume of the solid obtained by revolving R around the x -axis.



$$4 - x = \frac{3}{x} \Rightarrow 4x - x^2 = 3$$

$$\Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow x = 1 \text{ or } 3$$

$$\begin{aligned} 1. \text{ Area}(R) &= \int_1^3 \left(4 - x - \frac{3}{x} \right) dx = \left. 4x - \frac{1}{2}x^2 - 3 \ln x \right|_1^3 \\ &= 12 - \frac{9}{2} - 3 \ln 3 - \left(4 - \frac{1}{2} - 0 \right) = 8 - 4 - 3 \ln 3 \\ &= \boxed{4 - 3 \ln 3} \end{aligned}$$

$$\begin{aligned} 2. \text{ Vol} &= \pi \int_1^3 \left((4-x)^2 - \left(\frac{3}{x}\right)^2 \right) dx = \pi \int_1^3 \left(16 - 8x + x^2 - \frac{9}{x^2} \right) dx \\ &= \pi \left[16x - 4x^2 + \frac{x^3}{3} + \frac{9}{x} \right]_1^3 = \pi \left[48 - 36 + 9 + 3 - \left(16 - 4 + \frac{1}{3} + 9 \right) \right] \\ &= \pi \left[24 - 21\frac{1}{3} \right] = 2\frac{2}{3} \pi = \boxed{\frac{8\pi}{3}} \end{aligned}$$