R. Bruner
Math 2020, Winter 2007, Quiz 13
18 April 2007

1. If you know that $f^{(n)}(0) = 2^n n!$, what is the Taylor series for $f(x)$?
2. What is the radius of convergence of $f(x)$?
3. If $g(x) = \sum_0^\infty \frac{x^n}{(2n+1)!}$ find $g^{(n)}(0)$.
4. Find the radius of convergence of $g(x)$.

1. $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{2^n n!}{n!} x^n = \boxed{\sum_{n=0}^{\infty} 2^n x^n} = \frac{1}{1-2x}$

2. Ratio test: $\left| \frac{2^{n+1} x^{n+1}}{2^n x^n} \right| = |2x| < 1$ if $|x| < 1/2$
> 1 if $|x| > 1/2$

so $\boxed{R = 1/2}$

Root test is even easier: $\sqrt[n]{|2^n x^n|} = |2x|$.

3. $g(x) = \sum_0^\infty \frac{x^n}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{g^{(n)}(0)}{n!} x^n$ so

$\frac{g^{(n)}(0)}{n!} = \frac{1}{(2n+1)!}$ and we get $\boxed{g^{(n)}(0) = \frac{n!}{(2n+1)!}}$

4. Ratio test: $\left| \frac{\frac{x^{n+1}}{(2n+3)!}}{\frac{x^n}{(2n+1)!}} \right| = \left| \frac{x^{n+1}}{x^n} \frac{(2n+1)!}{(2n+3)!} \right|$

$= |x| \frac{1}{(2n+2)(2n+3)} \longrightarrow 0$ as $n \to \infty$ so $\boxed{R = \infty}$

(converges for all x)