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Math 2020, Winter 2007, Quiz 12
11 April 2007

Let $f(x) = \sum_{1}^{\infty} \frac{x^n}{n}$.

1. Find $f^{(n)}(0)$.

2. Find the power series for $f'(x)$.

3. What is the radius of convergence of $f(x)$?

1. $\frac{f^{(n)}(0)}{n!} = \frac{1}{n}$ so $f^{(n)}(0) = \frac{n!}{n} = (n-1)!$

2. $f'(x) = \sum_{1}^{\infty} \frac{n x^{n-1}}{n} = \sum_{1}^{\infty} \frac{x^{n-1}}{1} = \sum_{0}^{\infty} x^n$

3. Ratio test: $\left| \frac{x^{n+1}}{n+1} \Big/ \frac{x^n}{n} \right| = \left| \frac{x^{n+1}}{x^n} \frac{n}{n+1} \right|$

$$= |x| \left(\frac{n}{n+1}\right) \longrightarrow |x| = L$$

So $R = 1$ since the series converges absolutely if $L = |x| < 1$ and diverges if $L = |x| > 1$.