Solutions to some sample problems for Test 3 Math 2020, Winter 2007
R. Bruner,

1. (10) skip
2. (10) skip
3. (10) skip
4. skip
5. (25) skip
6. (10) It starts at $(x, y) = (1, 0)$, increases radius in the first quadrant until it gets to $(0, 2)$, then in the second quadrant, radius decreases until it hits $(0, 0)$, and then in the third quadrant radius increases again until it reaches $(0, -2)$, and finally in the fourth quadrant radius decreases and it returns to $(1, 0)$.
7. Consider the curve $x(t) = t^3 - 3t$, $y(t) = t^4 - 3t^2$.
   (a) (10) $dx/dt = 3t^2 - 3$ and $dy/dt = 4t^3 - 6t$.
   (b) (5) The curve is vertical where $x' = 0$, i.e., $t = \pm 1$.
   (c) (5) The curve is horizontal where $y' = 0$, i.e., $t = 0$ and $t = \pm\sqrt{3/2}$.
   (d) (10) The tangent line at $t = 1/\sqrt{3}$ is
   $$y + 8/9 = (-32/3\sqrt{3})(x + 8/3\sqrt{3})$$
   (e) (5) The integral which computes the length of the curve from $t = 0$ to $t = 1/\sqrt{3}$ is
   $$\int_0^{1/\sqrt{3}} \sqrt{(3t^2 - 3)^2 + (4t^3 - 6t)^2}\ dt$$
8. (10) The area inside the curve which is given in polar coordinates as $r = \sqrt{\cos 2\theta}$, $-\pi/4 \le \theta \le \pi/4$ is
   $$(1/2)\int_{-\pi/4}^{\pi/4} \cos 2\theta\ d\theta = (1/4)\sin 2\theta|_{-\pi/4}^{\pi/4} = 1/2.$$
9. (5) In polar coordinates $(x^2 + y^2)^2 = x^2 - y^2$ is $r^4 = r^2(\cos^2\theta - \sin^2\theta)$, or $r^2 = \cos^2\theta - \sin^2\theta$.
10. (5) In Cartesian coordinates $r\cos(\theta) = r^2\sin^2(\theta)$ is $x = y^2$.

——— The End ———