Some sample problems for Test 3 Math 2020, Winter 2007
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1. (10) Find the average value of x^2 on $[0, 2]$.

2. (10) Estimate $\int_1^3 \frac{dx}{x}$ using three equal intervals and evaluating the function at the midpoint of each interval.

3. (10) Let R be the region between $y = 1$ and $y = 2 - x^2$. (Hint: they intersect at $x = -1$ and $x = 1$.) Find the volume of the solid obtained by revolving R about the x-axis.

4. Find the centroid and area of the region in the preceding problem.

5. (25) Compute
   (a) $\int \frac{dx}{x^2-x-2}$
   (b) $\int \frac{x^2+2x}{x^2+4}\,dx$
   (c) $\int x \sin x\,dx$
   (d) $\int \sin^{10}(x)\cos(x)\,dx$
   (e) $\int_1^\infty \frac{dx}{\sqrt{x}}$

6. (10) Suppose that $f(\theta)$ increases from 1 to 2 as θ goes from 0 to $\pi/2$,
   - decreases to 0 as θ goes from $\pi/2$ to π,
   - increases to 2 as θ goes from π to $3\pi/2$, and
   - decreases to 1 as θ goes from $3\pi/2$ to 2π.

   Sketch the curve $r = f(\theta)$.

7. Consider the curve $x(t) = t^3 - 3t$, $y(t) = t^4 - 3t^2$.
   (a) (10) Compute dx/dt and dy/dt.
   (b) (5) Find both places where the curve is vertical.
   (c) (5) Find all three places where the curve is horizontal.
   (d) (10) Find the tangent line at $t = 1/\sqrt{3}$.
   (e) (5) Write the integral which computes the length of the curve from $t = 0$ to $t = 1/\sqrt{3}$. (You do not need to evaluate it.)

8. (10) Find the area inside the curve which is given in polar coordinates as $r = \sqrt{\cos 2\theta}$, $-\pi/4 \le \theta \le \pi/4$.

9. (5) Convert to polar coordinates $(x^2 + y^2)^2 = x^2 - y^2$.

10. (5) Convert to Cartesian coordinates $r\cos(\theta) = r^2\sin^2(\theta)$.

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