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**Math 2020, Fall 2016, Worksheet 3**  
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This worksheet concerns the Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

for several functions  $f(x)$  and their consequences.

1. Write the power series for  $f(x) = \frac{1}{1-x}$ .
  
  
  
  
  
  
  
  
  
  
2. What is its radius of convergence? What is its interval of convergence?
  
  
  
  
  
  
  
  
  
  
3. Substitute  $-x$  for  $x$  to get the power series for  $f(x) = \frac{1}{1+x}$ .
  
  
  
  
  
  
  
  
  
  
4. Integrate this term by term to get the Taylor series for  $\ln(1+x)$ .
  
  
  
  
  
  
  
  
  
  
5. What is its radius of convergence? What is its interval of convergence?

————— Continued on reverse —————

6. Substitute  $-x^2$  for  $x$  to get the power series for  $f(x) = \frac{1}{1+x^2}$ .

7. Integrate this term by term to get the Taylor series for  $\tan^{-1}(x)$ .

8. What is its radius of convergence? What is its interval of convergence?

9. Use this power series to describe the sum  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$ .

————— The End —————