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Math 2020, Fall 2016, Worksheet 2
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This worksheet concerns the Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \cdots + \frac{f^n(0)}{n!}x^n + \cdots$$

for several functions $f(x)$ and their consequences.

1. Write the power series for $f(x) = \dfrac{1}{1-x}$.

2. What is its radius of convergence? What is its interval of convergence?

3. Use the Taylor series formula above to deduce the value of $f^{(n)}(0)$.

4. Substitute $-x$ for x to get the power series for $f(x) = \dfrac{1}{1+x}$.

5. Integrate this term by term to get the Taylor series for $\ln(1+x)$.

6. Use this power series to describe the sum $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n\cdot 3^n}$.

7. If $f(x)=\sin(x)$ then $f(0)=0$, $f'(0)=1$, $f''(0)=0$, $f'''(0)=-1$, and $f^{n+4}(0)=f^n(0)$. Use these facts to write the Taylor series for $\sin(x)$.

8. What is its radius of convergence? What is its interval of convergence?

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