R. Bruner
Math 2020, Fall 2016, Worksheet 1
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1.

$$\int (2x-3)^9\,dx$$

Let $u = 2x-3$. Then $du = 2\,dx$, so $dx = \frac{1}{2}\,du$.

Then

$$\int (2x-3)^9\,dx = \int u^9 \cdot \frac{1}{2}\,du = \frac{1}{2}\int u^9\,du$$

$$= \frac{1}{2}\,\frac{1}{10}\,u^{10} + C = \boxed{\frac{1}{20}(2x-3)^{10} + C}$$

2.

$$\int \frac{\sin(x)}{1+\cos^2(x)}\, dx$$

Let $u = \cos(x)$. Then $du = -\sin(x)\,dx$, OR $-du = \sin x\, dx$.

Then we have $-\int \frac{du}{1+u^2} = -\tan^{-1}(u) + C$

$$\boxed{= -\tan^{-1}(\cos(x)) + C}$$

3.

$$\int \frac{\sin(2x)}{1+\cos^2(x)}\, dx$$

Recall $\sin(2x) = 2\sin x \cos x$.

This is then

$= \int \frac{2 \sin x \cos x \, dx}{1+\cos^2 x}$. If we let $u = \cos(x)$ as in #2, we get $= -\int \frac{2u}{1+u^2}\,du = -\ln|1+u^2| + C$

$$\boxed{= -\ln|1+\cos^2(x)| + C}$$

The substitution $u = 1+\cos^2(x)$ works even faster.

4.

$$\int \frac{1}{1+x^2}\,dx$$

Recall $\sin^2\theta + \cos^2\theta = 1$

gives $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

OR $\tan^2\theta + 1 = \sec^2\theta$.

So we let $x = \tan\theta$. Then $dx = \sec^2\theta\, d\theta$ so the integral becomes

$$\int \frac{\sec^2\theta\, d\theta}{1+\tan^2\theta} = \int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int d\theta = \theta + C = \boxed{\tan^{-1}(x) + C}$$

$[x = \tan\theta \iff \tan^{-1}x = \theta]$

5.

$$\int \frac{x}{1+x^2}\,dx$$

Here we let $u = x^2$ so that the integral becomes $\frac{1}{2}\int \frac{du}{1+u} = \frac{1}{2}\ln|1+u| + C$

$$= \boxed{\frac{1}{2}\ln|1+x^2| + C}$$

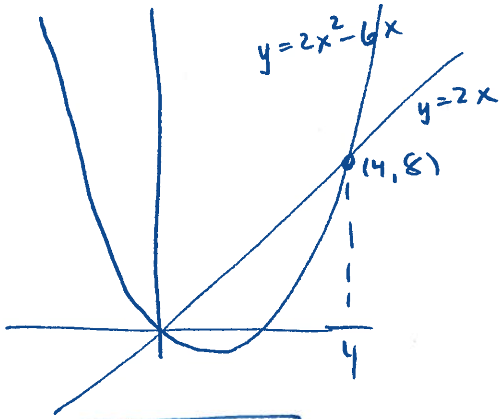
6.

$$\int \frac{x}{1+x^4}\,dx$$

Again use $u = x^2$, $du = 2x\,dx$, to get

$$= \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}(u) + C = \boxed{\frac{1}{2}\tan^{-1}(x^2) + C}$$

7. Describe the region between $y = 2x^2 - 6x$ and $y = 2x$ and find its area.



Region
$0 \le x \le 4$
$2x^2 - 6x \le y \le 2x$

Intersections: $2x^2 - 6x = 2x$

$\Rightarrow 2x^2 - 8x = 0$

$\Rightarrow 2x(x-4) = 0$

$\Rightarrow x = 0 \quad (0,0)$

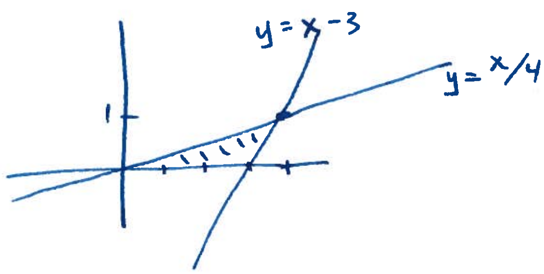
or $x = 4 \quad (4,8)$

$$\text{Area} = \int_0^4 2x - (2x^2 - 6x)\,dx = \int_0^4 8x - 2x^2\,dx$$

$$= 4x^2 - \frac{2}{3}x^3 \Big|_0^4 = 64 - 64 \cdot \frac{2}{3}$$

$$= \boxed{\frac{64}{3}}$$

8. Describe the region between the x-axis, $y = x - 3$ and $y = x/4$ and find its area.



Region:
$0 \le y \le 1$
$4y \le x \le y+3$

Intersections

$y=0$ & $y = x/4$
at $(0,0)$

$y=0$ & $y = x-3$
at $(3,0)$

$y = x/4$ & $y = x-3$
at $\frac{x}{4} = x - 3$
$x = 4x - 12$
$0 = 3x - 12$
$= 3(x-4)$

i.e. $x = 4, y = 1$

$$\text{Area} = \int_0^1 y + 3 - 4y \quad dy$$

$$= \int_0^1 3 - 3y\,dy = 3y - \frac{3}{2}y^2 \Big|_0^1$$

$$= 3 - \frac{3}{2} - (0 - 0) = \boxed{\frac{3}{2}}$$

OR $A = \frac{1}{2}bh = \frac{1}{2}(3)(1) = \frac{3}{2}$

——— The End ———