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Math 2020, Fall 2016, Test 3
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Write clearly, label your answers by problem number, and leave space between problems. You may keep this list of questions.

1. (5 each)
   (a) What is the graph of $x^2 + 1 = x^2$?
   (b) What is the graph of $(x + y)^2 = 2x + 2xy - 2y$?
   (c) Briefly explain what an integral is and what an antiderivative is.
   (d) Explain how integrals give antiderivatives (a simple but very important formula will help).
   (e) Explain how antiderivatives can be used to evaluate integrals. (Again, a simple formula will help.)
2. (10) Determine $\int_1^\infty \frac{dx}{x^7}$
3. (10) Determine $\int_{-1}^1 \frac{dx}{x^6}$
4. Consider the curve $x(t) = 1 - t^2$, $y(t) = t^3 - t$, for $-1 \le t \le 1$.
   (a) (5) Compute dx/dt and dy/dt.
   (b) (5) Compute dy/dx.
   (c) (5) Find those places where the curve is vertical.
   (d) (5) Find those places where the curve is horizontal.
   (e) (10) Compute the area inside the loop $-1 \le t \le 1$.
   (f) (5) Write the integral which computes the length of this curve.
5. (10) Find the area inside the curve which is given in polar coordinates as $r = \sin^2\theta$.
6. (5) Convert from polar to Cartesian coordinates: $r = \cos\theta$.
7. (5) Convert from Cartesian to polar coordinates: $x^2 - y^2 = 4$.
8. (10) Find the asymptotes of $x^2 + 10x + y^2 - 8y = 0$ and roughly sketch the curve.

——— Formulas on reverse ———

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{2}\sin x\cos x + C$$

$$\int \sin^4 x\,dx = \frac{3}{8}x - \frac{3}{8}\sin x\cos x - \frac{1}{4}\sin^3 x\cos x + C$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx + C$$

——— The End ———