Can we conclude that $\frac{\sqrt{n}}{n+1} > \frac{\sqrt{n+1}}{(n+1)+1}$

simply by observing that the denominator is increasing by more than the numerator?

No: $\frac{1}{2} \not> \frac{1+2}{2+3} = \frac{3}{5}$ even though the denominator increased by more than the numerator. Rates matter.

We can do this algebraically:

$$\frac{\sqrt{n}}{n+1} > \frac{\sqrt{n+1}}{n+2} \iff \frac{n}{(n+1)^2} > \frac{n+1}{(n+2)^2}$$ since both are positive

and this $\iff n(n+2)^2 > (n+1)^3$

$$\iff n(n^2+4n+4) > n^3+3n^2+3n+1$$

$$\iff n^3+4n^2+4n > n^3+3n^2+3n+1$$

$$\iff n^2+n > 1$$

so, since $n \in \{1,2,3,\dots\}$, this is true for all such n.