TEST 4 M2010 W'07 SOLUTIONS

10 each

1. (a) $\boxed{\tan(e^x) + \sqrt{x}}$

miss $3x^2$ (−3)

(b) $\boxed{3x^2 \cos(\sqrt{x^3}) - \cos\sqrt{x}}$

miss +C (−1)

(c) $$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = \boxed{2e^{\sqrt{x}} + C}$$

Let $u = \sqrt{x}$

so $du = \frac{1}{2\sqrt{x}}dx$

miss $\frac{1}{2}$ (−3)

(d) $$\int (2x-3)^9\,dx = \frac{1}{2}\int u^9\,du = \boxed{\frac{1}{2}\frac{(2x-3)^{10}}{10} + C}$$

Let $u = 2x-3$

so $du = 2\,dx$

3 pts for correct subst w/o further success

(e) $$\int_0^1 \frac{x}{x+1}\,dx = \int_1^2 \frac{u-1}{u}\,du = \int_1^2 1 - \frac{1}{u}\,du$$

$$= u - \ln|u| \Big|_1^2$$

Let $u = x+1$

so $du = dx$ and $x = u-1$

$$= 2 - \ln 2 - (1-0)$$

$$= \boxed{1 - \ln 2}$$

(f) $$\int_0^1 \frac{x+1}{x^2+1}\,dx = \int_0^1 \frac{x}{x^2+1}\,dx + \int_0^1 \frac{1}{x^2+1}\,dx$$

$$\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^2 \frac{du}{u} = \frac{1}{2}\ln 2$$

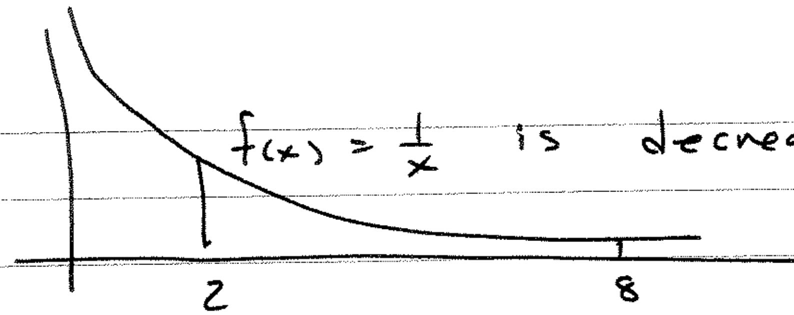
Let $u = x^2+1$

$\frac{1}{2}du = x\,dx$

$$\int_0^1 \frac{1}{x^2+1}\,dx = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$$

so $$\int_0^1 \frac{x+1}{x^2+1}\,dx = \boxed{\frac{1}{2}\ln 2 + \frac{\pi}{4}}$$

10

2. 

$f(x) = \frac{1}{x}$ is decreasing so
Left end = overest.
Right end = under.

$[2,8]$ broken into 3 gives $\Delta x = \frac{8-2}{3} = 2$

	Left	Right
$[2,4]$	$2 * \frac{1}{2}$	$2 * \frac{1}{4}$
$[4,6]$	$2 * \frac{1}{4}$	$2 * \frac{1}{6}$
$[6,8]$	$2 * \frac{1}{6}$	$2 * \frac{1}{8}$

-3 if reversed

Underest $= 2 * \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = \boxed{\frac{13}{12}}$

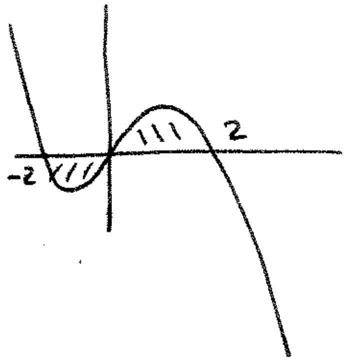
Overest $= 2 * \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right) = 1 + \frac{1}{2} + \frac{1}{3} = 1\frac{5}{6} = \boxed{\frac{11}{6}}$

5 3. $\int_2^5 f(x)\,dx = \int_0^5 f(x)\,dx - \int_0^2 f(x)\,dx = 7 - 5 = \boxed{2}$

5 4. $3 = \int_0^1 f(x) + 2g(x)\,dx = \int_0^1 f(x)\,dx + 2\int_0^1 g(x)\,dx = 7 + 2\int g$

so $\int_0^1 g(x)\,dx = (3-7)/2 = \boxed{-2}$

5. $y = 4x - x^3 = x(4-x^2) = x(2-x)(2+x)$



$$\text{Area} = \int_0^2 4x - x^3\, dx$$

$$= 2x^2 - \frac{1}{4}x^4 \Big|_0^2$$

$$= 2(4) - \frac{16}{4} - (0-0)$$

$$= 8 - 4 = \boxed{4}$$

6. $x \le f(x) \le 2x \Rightarrow \int_a^b x\, dx \le \int_a^b f(x)\, dx \le \int_a^b 2x\, dx$

(a) $\int_0^1 f(x)\, dx$ is between

$\int_0^1 x\,dx = \frac{1}{2}(1^2) = \frac{1}{2}$

and $\int_0^1 2x\,dx = 1^2 = 1$

(b) $F(x) = \int_0^x f(t)\, dt$ incr/decr $\overset{F'=f}{\Longleftrightarrow}$ f pos/neg.

Now $x \le f(x) \le 2x$ so f pos if $x > 0$ and f neg if $x < 0$.

So $\int_0^x f(t)\,dt$ is

incr on $(0, \infty)$

decr on $(-\infty, 0)$.

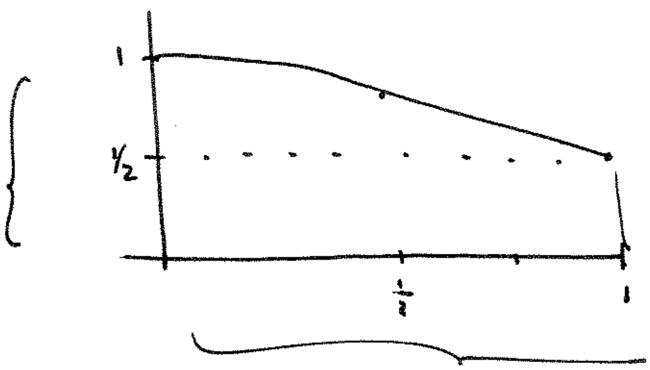
<u>Radians</u>, not degrees.

E.G. $\displaystyle\int_0^1 \cancel{450}\,\frac{1}{x^2+1}\,dx = \tan^{-1}(1) - \tan^{-1}(0)$

$\displaystyle\qquad = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

is approximately $\frac{3}{4}$ (radians) $= 45^\circ$. This is the area under $\dfrac{1}{x^2+1}$ for $0 \le x \le 1$

≤ height ≤ 1



width = 1

so the integral gives a value between $\frac{1}{2}$ and 1 (i.e. $\frac{\pi}{4} \simeq \frac{3}{4}$).

Certainly not 45 !!