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Let $f(x) = x^4 - 3x^2$.

1. Find all local maxima and minima of $f(x)$.
2. Find the absolute maximum and minimum values of $f(x)$ on the interval $[-2, 2]$.

$$f(x) = x^4 - 3x^2$$

$$f'(x) = 4x^3 - 6x = 2x(2x^2 - 3) \quad \text{defined for all } x.$$

1. At a local maximum or minimum, $f'(x) = 0$ since f' is defined everywhere. Now $f'(x) = 0$ is

$$2x(2x^2 - 3) = 0 \quad \text{so } x = 0 \quad \text{or } x^2 = \frac{3}{2}$$

Thus, at a local maximum or minimum we get

$$\begin{aligned} f(0) = 0 & \quad \text{or} \quad f\left(\pm\sqrt{\frac{3}{2}}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) \\ & = \frac{9}{4} - \frac{9}{2} = \frac{-9}{4} \end{aligned}$$

$\underbrace{\hspace{10em}}_{\text{local max}} \quad \underbrace{\hspace{10em}}_{\text{local min}}$
(2) (2, 2)

2. Values: $f(0) = 0$

$$f\left(\pm\sqrt{\frac{3}{2}}\right) = -\frac{9}{4}$$

$$f(-2) = 16 - 12 = 4$$

$$f(2) = 16 - 12 = 4$$

Maximum = 4
Minimum = $-\frac{9}{4}$

(2 ea.)