Solutions to Sample Test 4 : Version 1

$\boxed{1.}$ (a) $\frac{d}{dx}\int_1^x \sin(1-t^2)\,dt = \sin(1-x^2)$ 1st FTC

(b) $\frac{d}{dx}\int_{\sqrt{x}}^{x^2} \sin(1-t^2)\,dt = 2x\cdot\sin(1-x^4) - \frac{1}{2\sqrt{x}}\sin(1-x)$

(1st FTC and chain rule, or 2nd FTC and chain rule)

(c) $\frac{d}{dx}\int_3^4 \frac{1}{\sqrt{x}}\,dx = \frac{d}{dx}(\text{constant}) = 0$ But my notation was poor: x is both a free variable (in $\frac{d}{dx}$) and bound in $\int_3^4 \frac{1}{\sqrt{x}}dx$

(d) $\int \cos(1-3x)\,dx = -\frac{1}{3}\int \cos(u)\,du$

$u = 1-3x$ $\quad = -\frac{1}{3}\sin(u) + C$

$du = -3\,dx$

$-\frac{1}{3}du = dx$ $\quad = -\frac{1}{3}\sin(1-3x) + C$

(e) $\int e^{2x-1} + \frac{3}{x^2+1}\,dx = \frac{1}{2}e^{2x-1} + 3\tan^{-1}(x) + C$

(f) $\int (x-1)\sqrt{x}\,dx = \int x\sqrt{x} - \sqrt{x}\,dx = \int x^{3/2} - x^{1/2}\,dx = \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

(g) $\int_1^2 4x + \frac{1}{x}\,dx = 2x^2 + \ln|x|\Big|_1^2 = 8 + \ln 2 - (2 + \ln(1))$

$= 6 + \ln 2$

$\boxed{2}$ $\int_0^{10} x\,dx \le \int_0^{10} f(x)\,dx \le \int_0^{10} 2x\,dx$

$50 = \frac{1}{2}(10^2) \le \int_0^{10} f(x)\,dx \le 10^2 = 100$

3. $[1,3]$ broken into four subintervals has $\Delta x = \frac{3-1}{4} = \frac{1}{2}$

Left endpt estimate $= \frac{1}{2}\left(f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2})\right)$

$$= \frac{1}{2}\left(1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$= \frac{30+20+15+12}{60} = \frac{77}{60}$$

Right endpt estimate $= \frac{1}{2}\left(f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)\right)$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right) = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

$$= \frac{20+15+12+10}{60} = \frac{57}{60}$$

"Random" point estimate $= \frac{1}{2}\left(\frac{1}{1.37} + \frac{1}{1.72} + \frac{1}{2.15} + \frac{1}{2.91}\right)$

To answer the question properly any one of the answers above will suffice, as will any other Riemann sum using 4 subintervals.

4. $\int_1^2 f(x)\,dx = \int_1^5 f(x)\,dx - \int_2^5 f(x)\,dx = 10 - 8 = 2$

5. $\int_1^3 f(x) - 2g(x)\,dx = \int_1^3 f(x)\,dx - 2\int_1^3 g(x)\,dx = 7 - 2(4) = -1$

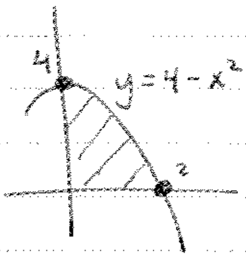
6. f increasing $\Rightarrow$ Left endpoints give an underestimate, and Right endpoints give an overestimate.

Δx_i	interval	underestimate	overestimate
3	$[0,3]$	$3 * f(0) = 3$	$3 * f(3) = 6$
2	$[3,5]$	$2 * f(3) = 4$	$2 * f(5) = 8$
3	$[5,8]$	$3 * f(5) = 12$	$3 * f(8) = 15$
2	$[8,10]$	$2 * f(8) = 10$	$2 * f(10) = 16$

x	$f(x)$
0	1
3	2
5	4
8	5
10	8

Underestimate $= 29$ Overestimate $= 45$

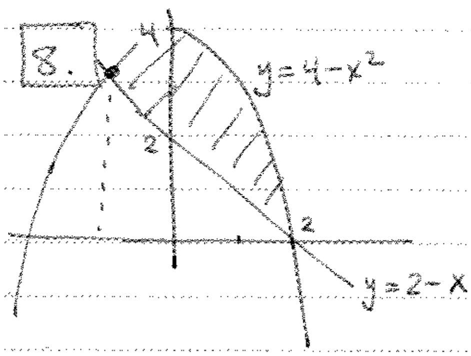
7.



$$\text{Area} = \int_0^2 4-x^2\,dx = 4x - \frac{x^3}{3}\Big|_0^2$$

$$= 8 - \frac{8}{3} - (0) = \boxed{\frac{16}{3}}$$

8.



$$\text{Area} = \int_{-1}^{2} (4-x^2) - (2-x)\,dx = \int_{-1}^{2} -x^2+x+2\,dx$$

$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2} = \frac{-8}{3} + 2 + 4 - \left(\frac{-1}{3} + \frac{1}{2} - 2\right)$$

$$= -\frac{7}{3} + 8 - \frac{1}{2} = \frac{-14+48-3}{6} = \boxed{\frac{31}{6}}$$

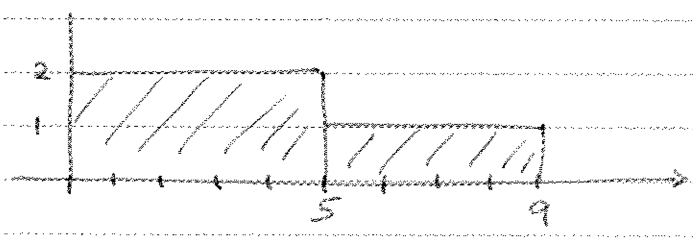
$$4 - x^2 = 2 - x$$
$$0 = x^2 - x - 2$$
$$0 = (x-2)(x+1)$$

So the line and parabola intersect at $x=-1$ and at $x=2$.

9.

$\int_1^5 f(x)\,dx = 8$ if $f(x) = 2$ on $[1,5]$

$\int_5^9 f(x)\,dx = \int_1^9 f(x)\,dx - \int_1^5 f(x)\,dx = 12 - 8 = 4$ if $f(x) = 1$ on $[5,9]$



There are an ∞ number of possible answers to this problem, of which this is one of the simplest.

Solutions to sample Test 4, version 2

1. (a) $\dfrac{d}{dx}\displaystyle\int_1^x \sqrt{2+\sin t}\,dt = \boxed{\sqrt{2+\sin x}}$

(b) $\dfrac{d}{dx}\displaystyle\int \sqrt{\frac{1}{4+e^x}}\,dx = \boxed{\sqrt{\frac{1}{4+e^x}}}$

(c) $\dfrac{d}{dx}\displaystyle\int_2^4 \frac{1}{t^2+1}\,dt = \boxed{0}$ (Derivative of a constant)

(d) $\displaystyle\int x e^{2x^2-3}\,dx = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \boxed{\frac{1}{4}e^{2x^2-3} + C}$

$u = 2x^2-3$

$du = 4x\,dx$

(e) $\displaystyle\int_1^2 \frac{x+1}{x}\,dx = \int_1^2 1+\frac{1}{x}\,dx = x+\ln|x|\Big]_1^2 = 2+\ln 2 - 1$

$= \boxed{1+\ln 2}$

(f) $\displaystyle\int_1^2 \frac{1}{x^2+1}\,dx = \boxed{\tan^{-1}(2) - \tan^{-1}(1)}$

2. $[0,2]$ broken into 4 has $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

Left endpoint estimate $= \boxed{\frac{1}{2}\left(\frac{1}{1^2+1} + \frac{1}{(\frac{3}{2})^2+1} + \frac{1}{2^2+1} + \frac{1}{(\frac{5}{2})^2+1}\right)}$

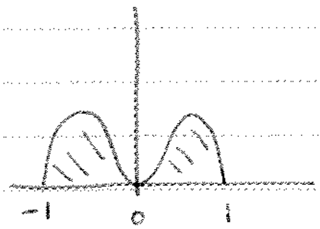
$= \frac{1}{2}\left(\frac{1}{2} + \frac{4}{13} + \frac{1}{5} + \frac{4}{29}\right)$

3. $\displaystyle\int_0^2 f(x)\,dx = \int_0^5 f(x)\,dx - \int_2^5 f(x)\,dx = 7 - 8 = \boxed{-1}$

4. $\displaystyle\int_0^3 g(x)\,dx = \int_0^3 2f(x)+g(x)\,dx - 2\int_0^3 f(x)\,dx$

$= 4 - 2(7) = \boxed{-10}$

$\boxed{5.}$ $y = x^2 - x^4 = x^2(1-x^2)$
$= x^2(1-x)(1+x)$



$$\text{Area} = \int_{-1}^{1} x^2 - x^4 \, dx$$
$$= \frac{1}{3}x^3 - \frac{1}{5}x^5 \Big|_{-1}^{1}$$
$$= \frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right)$$
$$= 2\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2(5-3)}{15}$$
$$= \boxed{\frac{4}{15}}$$

$\boxed{6.}$ $F(x) = \int_1^x t^2 - t \, dt$

F incr/decr $\iff$ F' pos/neg

so

$F'(x) = x^2 - x$

F incr on $(-\infty, 0)$ and $(1, \infty)$

F decr on $(0,1)$

$F''(x) = 2x - 1$

F conc up/dn $\iff$ F'' pos/neg

F conc up on $(\frac{1}{2}, \infty)$

conc dn on $(-\infty, \frac{1}{2})$

——— The End ———