

Solutions to Test 4 Math 2010 W'05

1. (a) $\frac{d}{dx} \int_1^x \sin(1-t^2) dt = \boxed{\sin(1-x^2)}$ (1st F.T.C.)

(b) $\frac{d}{dx} \int_3^4 \sqrt{x} dx = \boxed{0}$ (deriv. of a constant)

(c) $\int \cos(1-3x) dx = \left(-\frac{1}{3}\right) (\sin(1-3x)) + C$
 $= \boxed{-\frac{1}{3} \sin(1-3x) + C}$

(d) $\int (x-1)\sqrt{x} dx = \int x\sqrt{x} - \sqrt{x} dx = \int x^{3/2} - x^{1/2} dx$
 $= \boxed{\frac{2}{5} x^{5/2} - \frac{2}{3} x^{3/2} + C}$

(e) $\int_1^2 4x + \frac{1}{x} dx = 2x^2 + \ln|x| \Big|_1^2 = 8 + \ln 2 - (2 + \ln 1)$
 $= \boxed{6 + \ln 2}$

2. If $x \leq f(x) \leq 2x$ on $[0, 10]$ then

$$\int_0^{10} x dx \leq \int_0^{10} f(x) dx \leq \int_0^{10} 2x dx, \text{ so}$$

$$\frac{1}{2} x^2 \Big|_0^{10} \leq \int_0^{10} f(x) dx \leq x^2 \Big|_0^{10}, \text{ i.e.}$$

$$\boxed{50 \leq \int_0^{10} f(x) dx \leq 100}$$